Title and Abstract for Frances Kirwan Workshop

1. **Principle speaker**: Frances Clare Kirwan, University of Oxford, UK

Lecture 1: Non-reductive Geometric Invariant Theory

Abstract: Geometric invariant theory (GIT) was developed by Mumford in the 1960s in order to construct and studyquotients of algebraic varieties by actions of reductive linear algebraic groups. His main motivation was that many interesting moduli

Spaces in algebraic geometry can be constructed in this way. In general GIT for non- reductive linear algebraic group actions is much less well behaved than for reductive actions. However when the unipotent radical U of a linear algebraic group H is graded, in the sense that a Levi subgroup has a central one-parameter subgroup which acts by conjugation on U with all weights strictly positive, then GIT for a linear action of H on a projective scheme is almost as well behaved as in the reductive setting, provided that we are willing to multiply the linearisation by an appropriate rational character (joint work with Gergely Berczi, Brent Doran and Tom Hawes).

Lecture 2: Generalising symplectic implosion

Abstract: The symplectic reduction of a Hamiltonian action of a Lie group on a symplectic manifold plays the role of a quotient construction in symplectic geometry. It has been understood for several decades that symplectic reduction can be used to describe the quotients for complex reductive group actions in algebraic geometry provided by Mumford's GIT. There is an analogue of this description for GIT quotients by suitable non-reductive actions, which generalises the symplectic implosion construction of Guillemin, Jeffrey and Sjamaar. This involves a version of

a moment map and a Morse stratification provided by its norm-square, with applications including calculating Betti numbers and intersection pairings on non-reductive GIT quotients (joint work with Gergely Berczi).

Lecture 3: Moduli spaces of unstable objects

Abstract: Non-reductive GIT can be applied to the construction of moduli spaces in cases when classical GIT is not applicable. These include moduli spaces of 'unstable' objects of prescribed type, such as sheaves of fixed Harder-Narasimhan type, unstable projective curves or projective schemes of dimension greater than 1 (joint work with Gergely Berczi, Vicky Hoskins and Joshua Jackson).

2. Bohan Fang, BICMR, Peking University,China

Title: Oscillatory Integrals and Gamma II Conjecture

Abstract: The mirror of a complete toric Fano variety is a Landau-Ginzburg model. Oscillatory integrals over these mirror branes will give genus 0 Gromov-Witten descen- dant potentials. By identifying Lefschetz thimbles with T-dual branes, we can show that these integrals have 1) desired asymptotic expansions by stationary phase ex-pansion;

2) correspond to descendant potentials with I ritani's Gamma-classes. Thus this comes to the Gamma I I conjecture f or toric Fano varieties, which asserts f or a Fano variety the asymptotic solutions to the quantum differential equations do have analytic l ifts corresponding to the Gamma-classes of an exceptional collection.

3. Lothar G¨ottsche, International Center for theoretical Physics, Italy

Title: Virtual Refinements of the Vafa-Witten Formula

Abstract: Vafa and Witten made predictions about the Euler numbers of moduli spaces of sheaves on surfaces. They give explicit generating functions in terms of modular forms. These moduli spaces are in general very singular, but they have a perfect obstruction theory, and thus a virtual fundamental class and a virtual Tangent bundle, and thus virtual Chern numbers and in particular a virtual Euler number. We interpret the prediction as being about the virtual Euler numbers. Then a formula of Mochizuki allows to compute the virtual Euler numbers in terms of integrals on Hilbert schemes of points, which we do via reduction to toric surfaces and virtual localization. This allows to check the conjecture in a wide variety of cases up to high expected dimensions of the moduli spaces. We then extend the conjecture first to the χ_y-genus and then to the elliptic genus, where we obtain generating functions similar to that of Dijkgraaf-Moore-Verlinde-Verlinde for Hilbert schemes of points. Finally we extend the conjectures to the virtual cobordism class of the moduli spaces.

4. Jochen Heinloth, University at Duisburg-Essen, Germany

Title: Existence of Moduli Spaces for Algebraic Stacks

Abstract: Recently Alper, Hall and Rydh gave general criteria when a moduli problem can locally be described as a quotient and thereby clarified the local structure of algebraic stacks. We report on a joint project with Jarod Alper and Daniel Halpern-Leistner in which we use these results to show general existence results for good coarse moduli spaces. In the talk we will focus on one aspect that illustrates how the geometry of algebraic stacks gives a new point of view on classical methods, namely we explain how Langton's proof of semistable reduction for coherent sheaves on projective varieties can be reformulated in terms of geometry. This allows to prove semistable reduction for an interesting class of moduli problems.

5. David Hyeon, Marshall University, USA

Title: Toward a GIT Construction for a Moduli Space of Commuting Nilpotents

Abstract: I will describe how a moduli space of commuting nilpotents may be constructed via GIT and how non-reductive GIT can make things much simpler.

6. Young-Hoon Kiem, Seoul National University, Korea

Title: 30 Years of Partial Desingularization

Abstract: Geometric invariant theory (GIT) quotients of smooth projective varieties are often singular. By Luna's slice theorem, the singularities arise from non-trivial stabilizers and often bigger stabilizers result in worse singularities. In 1985, Frances Kirwan invented an algorithm, called the partial desingularization pocess, that systematically resolves all the singularities worse than orbifold singularities by a sequence of blowups. In this talk, I'd like to discuss applications of the partial desingularization process during the past 30 years, in the theory of moduli of vector bundles on curves, in birational geometry of moduli spaces, and in the construction of symplectic varieties. Finally, I'd also like to talk about a recent joint work with Jun Li and Michail Savvas about a theory of generalized Donaldson-Thomas invariant by partial desingularization.

7. Yoshinori Namikawa, University of Kyoto, Japan

Title: Poisson Deformations and Birational Geometry

Abstract: The semiuniversal deformation of a Klein singularity was constructed by Grothendieck, Brieskorn and Slodowy and the Weyl group of a complex Lie algebra naturally appears in its simultnaneous resolution. We generalize these results to conical symplectic varieties by using Poisson deformations. We will also discuss birational geometry related to Poisson deformations.