Title and abstract for The Legacy of Carl Friedrich Gauss

1. Principle Speaker: Shing-Tung Yau, Harvard University, USA

Lecture 1: My past experience in mathematics

Abstract: Studying mathematics is like writing a novel where characters and plots needed to remain realistic. The cultivation of emotions is the most important. Excellent mathematics should also touch on various phenomena in nature to be passed down for generations. I will talk about my experience in mathematics since childhood.

Lecture 2: Calabi-Yau Varieties and the SYZ Congecture

2. Huaidong Cao, Lehigh University, USA

Title: Deformations of Fano Manifolds

Abstract: For an analytic family of small deformations of a Fano manifold, Koiso in mid 1980s proved if the central fiber is a Fano Kähler-Einstein (KE) manifold which does not admit any nontrivial holomorphic vector fields then each (nearby) fibers also admit a KE metric. In this talk, we shall present a new necessary and sufficient condition on the existence of KE metrics on small deformations of a Fano KE manifold with nontrivial automorphism group. If time permits, we will also describe a canonical extension of pluri-anticanonical forms from a Fano KE manifold to its small deformations which leads to a simultaneous embedding of a family of Fano manifolds into projective spaces with effective control. This is a joint work with Xiaofeng Sun, S.-T. Yau and Yingying Zhang.

3. Gavril Farkas, University of Berlin, Germany

Title: Fundamental Groups, Alexander Invariants and Syzygies of Canonical Curves

Abstract: I will discuss a purely algebraic statement concerning an explicit descrip- tion of the Chow form of the Grassmannian of lines in projective space. This algebraic statement, which turns out to be equivalent to Mark Green's Conjecture on syzygies of canonical curves (proven by Claire Voisin), has many interesting topological appli- cations of which I will discuss (1) a universal upper bound on the nilpotence index of the fundamental group of any compact Kaehler manifold and (2) a bound on the length of the nilpotence index on the Torelli groups associated to the moduli space of curves. This is joint work with Aprodu, Papadima, Raicu and Weyman.

4. Maksym Fedorchuk, Boston College, USA

Title: Invariant-theoretic Mather-Yau Theorem, and Applications

Abstract: The famous Mather-Yau theorem says that two isolated hypersurface sin- gularities of the same embedding dimension are biholomorphically equivalent if and only if their moduli algebras are isomorphic. Thus determining whether two giv- en moduli algebras are isomorphic becomes an important problem. In the case of quasi-homogeneous hypersurface singularities, Eastwood and Isaev proposed a purely algebraic approach to this problem, rooted in classical invariant theory. For homoge- neous singularities, this approach leads to the associated form morphism that assigns to a singularity the Macaulay inverse system of its moduli algebra. The associated form morphism has several marvelous properties, two of which I will discuss in this talk. In joint work with Isaev, we have shown that the associated form morphis- m preserves GIT polystability (arXiv:1703.00438). This leads to a purely algebraic invariant-theoretic Mather-Yau theorem for homogeneous hypersurface singularities,

and to interesting new compactifications of the moduli space of smooth hypersur- faces. The associated form morphism also detects whether a homogeneous polyno- mial with a non-vanishing discriminant is a direct sum (of Sebastiani-Thom type), and so leads to an algorithm for finding direct sum decompositions over the rationals (arXiv:1705.03452).

5. Xianghong Gong, University of Wisconsin-Madison, USA

Title: Hölder estimates for homotopy operators on strictly pseudoconvex domains with C^2 boundary

Abstract: We derive a new homotopy formula for a strictly pseudoconvex domain of C^2 boundary in $\mathbf{C}^n$ by using a method of Lieb and Range and obtain estimates in Lipschitz spaces for the homotopy operators. For $r > 1$ and $q > 0$, we obtain a $\Lambda_{r+1/2}$ solution u to $\bar\partial u = f$ for a $\bar\partial$-closed $(0, q)$ form f of class Λ_r in the domain.
We apply the estimates to obtain boundary regularities of D-solutions for a domain in the Levi-flat Euclidean space.

6. Bo Guan, Ohio State University, USA

Title: Subsolutions and Concavity in Second Order Estimates for Fully Nonlinear PDEs on Real and Complex Manifolds

Abstract: Fully nonlinear elliptic and parabolic equations on manifolds play central roles in some important problems in real and complex geometry. A key ingredient in solving such equations is to establish apriori estimates up to second order. For general Riemannian manifolds, or Kähler/Hermitian manifolds in the complex case, one encounters difficulties caused by the curvature (as well as torsion in the Hermian case) of the manifolds. In this talk we report some results in our effort to overcome these obstacles over the past few years. We shall emphasize on understanding the roles of subsolutions and concavity of the equation based on which our techniques were developed. We are interested both in equations on closed manifolds, and in the Dirichlet problem for equations on manifolds with boundary, without imposing any restrictions to the geometry of the boundary.

7. Shinobu Hosono, Gakushuin University, Japan

Title: GKZ Hypergeometric Systems in Mirror Symmetry of Complete Intersection Calabi-Yau Manifolds
Abstract: Since the discovery of mirror symmetry of Calabi-Yau manifolds in 90's, Gel'fand-Kapranov-Zelevinski (GKZ) hypergeometric systems have been playing a central role to explore, and describe explicitly, mathematical consequences from the mirror symmetry. In this talk, I will focus on some interesting examples of Calabi-Yau threefolds of complete intersections and calculate the global monodromy of the GKZ systems for period integrals. In these examples, I will find that the monodromy nilpo- tent cones, which are defined locally for each boundary point of some distinguished properties, are naturally glued together by global monodromy relations coming from boundary divisors. Also I will find nice correspondences between the gluing of nilpo- tent cones and the movable cones in the birational geometry of mirror Calabi-Yau manifolds.

This talk is based on my recent collaboration with Hiromichi Takagi, *Movable vs monodromy nilpotent cones of Calabi-Yau manifolds* (arXiv:1707.08728), which goes back to my collaborations with A. Klemm, B. Lian, S. Theisen and S.-T. Yau in 90's.

8. Jiaxin Hu, Tsinghua University, China

Title: Heat Kernel Estimates: Gaussian and non-Gaussian

Abstract: I will give a short survey on heat kernel estimates for regular Dirichlet forms on metric measure spaces. For a local Dirichlet from, the heat kernel admits sub-Gaussian or Gaussian estimates, whilst for a nonlocal Dirichlet from, the heat kernel admits stable-like estimates.

9. Naoki Imai, University of Tokyo, Japan

Title: Non-semi-stable Loci in Hecke Stacks and Fargues' Conjecture

Abstract: Recently, a geometrization of the local Langlands correspondence is pro- posed by Fargues. In this talk, we discuss a non-semi-stable locus in a Hecke stack, which appears in the geometrization of the local Langlands correspondence. We find that a generalization of a diamond of a non-basic Rapoport-Zink space at infinite level covers the non-semi-stable locus, and show the Harris-Viehmann conjecture for this space under some HN-reducibility condition. As an application, we show the Hecke eigensheaf property in Fargues' conjecture for cuspidal Langlands parameters in the $GL(2)$-case. This is a joint work with Ildar Gaisin.

10. Xiaohuan Mo, Peking University, China

Title: On Finsler Surfaces of Constant FlagCurvature

Abstract: We discuss Finsler surfaces of constant (flag) curvature. First, we show that the space of those with two dimensional isometric group depends on two arbitrary constants. We also give a new technique to recover Finsler metrics from the specified two constants. Using this technique we obtain some new Finsler surfaces of constant flag curvature with two dimensional isometric group. Then we show that the space of Finsler metrics with constant flag curvature of which admits a Killing field depends on two arbitrary functions of one variable. Furthermore we find an approach to calculate these functions for spherically symmetric Finsler surfaces of constant flag curvature. In particular, we obtain the normal form of the Funk metric on the unit disk D^2.

11. Yoshinori Namikawa, University of Kyoto, Japan

Title: Towards the Classification of Symplectic Singularities

Abstract: After introducing the finiteness theorem for symplectic singularities, I will give a characterisation of nilpotent orbit closures of a complex semisimple Lie algebra.

10. Rita Pardini, Universita` di Pisa, Italy

Title: Linear systems on irregular varieties

Abstract: I will report on joint work M.A. Barja (UPC, Barcelona) and L. Stoppino (Universit`adell'Insubria, Como – Italy).

Given a generically finite map $a : X \to A$, where X is a smooth projective variety and A is an abelian variety, and given a line bundle L on X, we study the linear system $|L \otimes P|$, where P is a general element of $\text{Pic}^0(A)$. We prove that up to taking base change with a suitable multiplication map $A \to A$, the map given by $|L \otimes P|$ is independent of P and induces a factorization of the map a. When L is the canonical bundle of X, this factorization is a new geometrical object intrinsically attached to the variety X.

The factorization theorem also allows us to improve, under certain assumptions on the map $a : X \to A$, the known Clifford-Severi and Castelnuovo type numerical inequalities for line bundles on X. A key tool in these proofs is the introduction of a real function, the continuouscontinuousrank function, that also allowsustosimplify considerably the proof of the Clifford-Severi inequalities.

12. Lianfen Qian, Florida Atlantic University, USA

Title: Analysis of Spatial Correlation and Regional Finance in China via Network Analysis

Abstract: Using the data consisting the total of lending and deposit balances, and gross domestic product of 31 provinces in China from 1990 to 2015, this paper utilizes the financial location entropy as a measure of regional financial development indicator to construct the spatial correlation networkanalyses for China'sfinancial development afterGranger causality tests. The paper also studies the influential factors of China's regional financial development based on block models and QAP method. It effectively resolves the difficulty that the traditional spatial method faces when analyzing the financial linkage on the global characteristics of network structure. The empirical results show that: (1) the spatial correlation of China's regional financial development is complex, but the network is stability and has good accessibility. (2) China's regional financial development is divided into four sectors. The first sector is the main benefit plate in the less developed areas in the west, and the second plate is the agent plate, which plays the role of bridge. The third plate is the net overflow plate in the eastern regions, and the fourth plate is two-way overflow plate, mainly in the more developed provinces in the middle east of China. This indicates that China's regional financial development has clear energy transfer gradient characteristics. (3) Per capita income level, foreign trade index, the index of the third industry development, the influence of the transport ability and the ability of government intervention are important factors, while regional employment situation and the location have little influence on adjacent provinces. Other conclusions and suggestions are also presented.

13. Takeshi Saito, University of Tokyo, Japan

Title: Characteristic Cycle of an l-adic Sheaf

Abstract: The characteristic cycle of an f-adic sheaf on a smooth variety over a perfect field is a $\mathbf{Z}$-linear combination of irreducible components of the singular sup- port, defined by Beilinson as a closed conical subset of the cotangent bundle. It is an algebraic analogue of that studied by Kashiwara and Schapira in a transcendental setting. We discuss its functorial property with respect to proper direct image.

14. Xiaotao Sun, Chinese Academy?of Sciences, China

Title: A Finite Dimensional Proof of Verlinde Formula

Abstract: By degenerating a smooth curve to an curve with one node (irreducible or reducible), we establish two recurrence relations for the dimensions of spaces of generalized theta functions on moduli spaces of semi-stable parabolic bundles on smooth curves of genus g, which imply an explicit formula of dimension (Verlinde formula).

There are two steps to establish such recurrence relations: (1) factorizations of generalized theta functions over nodal curves;(2) invariance of dimensions during de-generation, which are implied by vanishing theorem of cohomolgy on moduli spaces. The step (1) and step (2) for $g > 2$ were done by myself around 2000. However vanishing theorem for $g < 3$ remains open. Recently, we prove that moduli spaces of semi-stable parabolic bundles and generalized parabolic sheaves with fixed determi- nants are of globally Frobenius regular type, which implythe vanishing theorem for any genus.

15. Yuichiro Taguchi, Tokyo Institute of Technology, Japan

Title: On the Moduli of Galois Representations

Abstract: We explain the construction of a certain moduli space of Galois represen- tations. More generally, for an object in a rather general class A of non-commutative topological rings, we construct a moduli space of its absolutely irreducible repre-sentations of a fixed degree as a (so we call) "f-A scheme". Various problems on Galois representations can be reformulated in terms of such moduli schemes. As an application, we show that the "difference" between the strong and weak versions of the finiteness conjecture of Fontaine-Mazur is filled in bythe finiteness conjecture of Khare-Moon.

16. Alessandro Verra, University of Rome, Italy

Title: K3 Surfaces and Moduli of E´taleCyclic Covers of Curves

Abstract: Complex K3 surfaces S are considered, suitably polarized in genus g by $H \in$ Pic S, which are quotients of K3 surfaces $\tilde{S}$ by a symplectic automorphism of order n. Let $P_{g,n}$ be the moduli space of triples (S,H,C), with $C \in |H|$, and let $R_{g,n}$

be the moduli space of degree n, cyclic ´etale coverings $\tilde{C} \to C$ of curves of genus g. Then (S,H,C) defines a covering $\tilde{C} \to C$, induced by the quotient map $\tilde{S} \to S$.

The assignement $(S, H, C) \to (\tilde{C} \to C)$ defines a map $r_{g,n} : P_{g,n} \to R_{g,n}$. This is a variation of the Mukai map $m_g : P_g \to M_g$ from the moduli of triples (S,H,C), where (S, H) is any polarized K3 surface polarized in genus g, to the moduli of curves of genus g.

In the talk some unexpected and remarkable analogies between the two maps are described. The cases $n = 2, 3$ are considered with more detail. For $n = 3$ the standard irreducible component of $P_{g,n}$ is studied, focusing on the transition case $g = 5$. The unirationality of $R_{5,3}$ is proven via the map $r_{5,3}$. For n = 2 the generic injectivity of $r_{g,2}$, with its exceptions and analogies to m_g, is outlined. (Joint works with A. Garbagnati and with A. Knutsen, M. LelliChiesa).

17. Angelo Vistoli, The Advanced Normal School of Pisa, Italy

Title: Chow Rings of Some Moduli Spaces of Smooth Curves

Abstract: There is by now an extensive theory of rational Chow rings of moduli spaces of smooth curves. The integral version of these Chow rings is not as well understood. I will survey what is known, including some recent developments.

18. Fengyu Wang, Center of Applied Mathematics, Tianjin University, China

Title: Coupling by Change of Measures and Applications

Abstract: In order to establish infinite-dimensional Harnack inequality for Markov semigroups we introduce the method of "Coupling by change of measures". This method is also applied to investigate Bismut type derivative formulas, integration by parts formulas, shift Harnack inequalities and applications. A simple example of SDE is considered to explain the main idea of the method.

19. Hongwei Xu, Zhejiang University, China

Title: Stronger Version of Chern Conjecture for Minimal Hypersurfaces

Abstract: The famous Chern Conjecture for minimal hypersurfaces with constant scalar curvature in a sphere was proposed by S.-S. Chern around 1970, and was listed in the Problem Section by S.-T. Yau in 1982. In this talk, I will talk about recent progress on the stronger version of the Chern Conjecture and its related problems. Based on the previous work due to Peng-Terng [Math. Ann., 1983], Ding-Xin [Adv. Math., 2011] and Xu-Xu [J. Funct. Anal., 2017], we prove that if M is a compact minimal hypersurface in S^{n+1} whose squared length of the second fundamental form satisfying $0 \leq S\text{-}n \leq n/18$, then $S = n$ and M is a Clifford torus. This is a joint work with Dr. Li Lei and Dr. Zhiyuan Xu.

20. Jae-Hyun Yang, Inha University, Korea

Title: The Stability of Automorphic Forms and Its Geometric Applications

Abstract: In this talk, I will introduce the notion of the stability of automorphic forms, and apply the stability of automorphic forms to the study of the moduli space of abelian varieties, the moduli space of curves, the moduli space of polarized real tori, the universal family of abelian varieties and the universal family of polarized real tori.

21. Lixin Zhang, Zhejiang University, China

Title: Adaptive Randomization: Efficiency, Selection Bias and Randomization Meth- ods

Abstract: Efrons (1971) biased coin design and Pocock and Simon's (1971) proce- dure are extensively implemented for balancing treatment allocation and balancing treatment allocation over influential covariates in clinical trials. However, the theoret- ical properties of the power of the conventional testing hypotheses and its relationship with the selection bias are usually unknown. In the literature, most studies are based on simulations. In this talk, we will consider the asymptotic relative loss of power of hypothesis testing to compare the treatment effects and the asymptotic selection bias under covariate-adaptive randomization procedures. We propose a new framework of covariate-adaptive design and establish the corresponding asymptotic theorems under widely satisfied conditions. A new covariate-adaptive design is defined by choosing suitable allocation function so that the selection bias is asymptotically the same as the complete randomization and the treatment imbalances considered are of the order
of $o(n^{1/2})$ in probability for which the loss of power is asymptotically ignorable.

22. Youjin Zhang, Tsinghua University, China

Title: Hodge Integrals and Integrable Systems

Abstract: For an arbitrary semisimple Frobenius manifold we construct an inte- grable hierarchy of Hamiltonian partial differential equations. In the particular case of quantum cohomology the tau-function of a solution to the hierarchy generates the intersection numbers of the Gromov–Witten classes and their descendents along with the characteristic classes of Hodge bundles on the moduli spaces of stable maps. By considering the integrable hierarchies associated to the one-dimensional Frobe- nius manifold, we establish a correspondence between a certain class of special cubic Hodge integrals and the so called fractional Volterra hierarchy.

23. Jian Zhou, Tsinghua University, China

Title: Gauss and Elliptic Functions

Abstract: In modern mathematical physics and number theory, elliptic functions and modular forms are widely used. By reading some manuscripts of Gauss, we have found that many results named after other mathematicians were actually first found by Gauss. They include: The notion of elliptic functions, theta functions, Fourier transform and Poisson summation formula, Jacobi triple product identity, modular group, fundamental domain, analogue of Klein J-function, etc. He also discovered the relation to hypergeoemtric equation and defined mirror map widely used in mirror symmetry. In this talk we will give a sampling of such results of Gauss obtained while he was also working on his Disquisitiones Arithmeticae.