TITLE AND ABSTRACTS

Shibananda Biswas, Indian Institute of Science Education and Research, Kolkata, India

Title: On homogeneous operators via quotient modules

Abstract: We show that the quotient modules obtained from submodules, consisting of functions vanishing to order k along a linear variety of codimension at least 2, of an analytic Hilbert module, is homogeneous with respect to suitable subgroup of the automorphism group of the domain. We are then able to show that these homogeneous operators are reducible and decomposes to generalized Wilkin's operators as irreducible components.

This talk is based on joint work with Prahllad Deb and Subrata Shyam Roy.

Monojit Bhattacharjee, Indian Institute of Technology, Bombay, India

Title: Factors of Hypercontractions

Abstract: In this talk, we discuss about a class of contractive factors of m-hypercontractions for $m \in \mathbb{N}$. We introduce a characterization of such factors and this is achieved by finding explicit dilation of these factors on some weighted Bergman spaces. This is a generalization of the work done in Factorizations of Contractions by Das, Sarkar and Sarkar.

This talk is based on joint work with B. Krishna Das.

%textbfGuangfu Cao, South China Agricultural University, China

$$S_\varphi F(z) = \frac{1}{\pi^n} \int_{\mathbb{C}^n} F(w) e^{z\cdot\bar{w}} \varphi(z-\bar{w}) e^{-|w|^2} dw, \qquad z \in \mathbb{C}^n,$$

is bounded on $\mathcal{F}^2(\mathbb{C}^n)$ if and only if there exists a function $m \in L^\infty(\mathbb{R}^n)$ such that

$$\varphi(z) = \int_{\mathbb{R}^n} m(x) e^{-2\left(x-\frac{i}{2}z\right)\cdot\left(x-\frac{i}{2}z\right)} dx, \qquad z \in \mathbb{C}^n.$$

With this characterisation we are able to obtain some fundamental results including the normaility, the algebraic property, spectrum and compactness of this operator S_φ. Moreover, we also obtain the reducing subspaces of S_φ.

In particular, in the case $n = 1$, we give a complete solution to an open problem proposed by K. Zhu for the Fock space $\mathcal{F}^2(\mathbb{C})$ (Integr. Equ. Oper. Theory **81** (2015), 451–454).

Lijia Ding, Peking University, China

Title: The $L^p - L^q$ Problems of Bergman-type Operators

Abstract: Let $\mathbb{B}^d$ be the unit ball on the complex space $\mathbb{C}^d$ with normalized Lebesgue measure dv. For $\alpha \in \mathbb{R}$, the Bergman-type integral operator K_α on $L^1(\mathbb{B}^d, dv)$ is defined by

$$K_\alpha f(z) = \int_{\mathbb{B}^d} \frac{f(w)}{(1 - \langle z, w\rangle)^\alpha} dv(w).$$

It is an important class of operators in the holomorphic function space theory over the unit ball. We also consider the integral operator K_α^+ on $L^1(\mathbb{B}^d, dv)$ which is given by

$$K_\alpha^+ f(z) = \int_{\mathbb{B}^d} \frac{f(w)}{|1 - \langle z, w\rangle|^\alpha} dv(w).$$

In this talk, we mainly concern the L^p-L^q boundedness and compactness of K_α, K_α^+. We will give some equivalent characterization of the L^p-L^q boundedness of K_α, K_α^+ and compactness of K_α. The results of boundedness are in fact the Hardy-Littlewood-Sobolev type theorem but also prove the conjecture of [G. Cheng et al, Trans. AMS, 2017] in the case of bounded domain $\mathbb{B}^d$. Meanwhile, a trace formula and some sharp norm estimates of K_α, K_α^+ are given.

Yongjiang Duan, Northeast Normal University, China

Title: Toeplitz operators on weighted harmonic Bergman spaces

Abstract: We first introduce Condition (L) and (B) to describe the bounded Toeplitz operator T_g with integrable symbols on the regular weighted Bergman space $L_a^1(\omega)$. In addition, we discuss the boundedness and compactness of the Toeplitz operator T_μ from $L_h^p(\omega)$ to $L_h^q(\omega)$ for $1 < p, q < \infty$, where μ is a positive Borel measure.This is joint work with Kunyu Guo, Siyu Wang and Zipeng Wang.

Xiang Fang, National Central University, Taiwan, China

Title: Regularity of Random Analytic Functions

Abstract: Recently we are trying to understand what random operator theory (ROT) means, and we quickly realize that one needs to understand random analytic functions first. For random Hardy functions, there is a classical theorem of Littlewood in 1930. We will present improvements and variants of it. For the Bergman space, nothing is known (as far as our knowledge is concerned), and we will prove a Bergman version of Littlewoods theorem. The Dirichlet space will be discussed as well. There are many open problems in this area. (Joint work with Cheng Guozheng)

Soumitra Ghara, Indian Institute of Science, Bangalore, India

Title: Decomposition of the tensor product of two Hilbert modules

Abstract: Given a pair of positive real numbers α, β and a sesqui-analytic function K on a bounded domain $\Omega \subset \mathbb{C}^m$, in this talk, we will investigate the properties of the sesqui-analytic function

$$\mathbb{K}^{(\alpha,\beta)} := K^{\alpha+\beta}\big(\partial_i \bar{\partial}_j \log K\big)_{i,j=1}^{m}$$

taking values in $m \times m$ matrices. One of the key findings is that $\mathbb{K}^{(\alpha,\beta)}$ is non-negative definite whenever K^α and K^β are non-negative definite. In this case, a realization of the Hilbert module determined by the kernel $\mathbb{K}^{(\alpha,\beta)}$ will be discussed. Let $\mathcal{M}_i$, $i = 1, 2$, be two Hilbert modules over the polynomial ring $\mathbb{C}[z_1, \ldots, z_m]$. Then $\mathbb{C}[z_1, \ldots, z_{2m}]$ acts naturally on the tensor product $\mathcal{M}_1 \otimes \mathcal{M}_2$. The restriction of this action to the polynomial ring $\mathbb{C}[z_1, \ldots, z_m]$ obtained using the restriction map $p \mapsto p_{|\Delta}$ leads to a natural decomposition of the tensor product $\mathcal{M}_1 \otimes \mathcal{M}_2$, which is investigated. Two of the initial pieces in this decomposition are identified.

This talk is based on joint work with Gadadhar Misra.

Kunyu Guo, Fudan University, China

Title: The Kozlov completeness problem

Abstract: The classical completeness problem raised by Beurling(1945) and independently by Wintner(1944-1945) asks for which $\psi \in L^2(0,1)$, the dilation system $\{\psi(kx) : k = 1, 2, \cdots\}$ is complete in $L^2(0,1)$, where ψ is identified with

its odd 2-periodic extension on $\mathbb{R}$. This difficult problem is nowadays commonly called as the Periodic Dilation Completeness Problem (PDCP). The PDCP has a natural link with the famous Riemann Hypothesis. Since the set of simple functions on $(0,1)$ is dense in $L^2(0,1)$, ones focus on the above problem for the class of characteristic functions. Let χ_s be the characteristic function of $(0,s)$, $0 < s \leq 1$, and $\mathcal{D}_s = \{\chi_s(kx) : k = 1, 2, \cdots\}$. The Kozlov completeness problem is to ask for which s, the dilation system $\mathcal{D}_s$ is complete(1948-1950). In this talk, I will give a brief introduction for what we have made some significant progress on the PDCP and the Kozlov problem. This is a joint work with Dr.Dan.

Gargi Ghosh, Indian Institute of Science Education and Research, Kolkata, India

Title: An extension of the Chevalley-Shephard-Todd Theorem

Abstract: For any $f \in \mathbb{A}^2(\mathbb{D}^2)$, we can express f as a linear combination of 1 and $z_1 - z_2$ with $\mathfrak{S}_2$-invariant coefficients. Thus we split the space $\mathbb{A}^2(\mathbb{D}^2)$ into space of symmetric functions and anti-symmetric functions and we realize each of these subspaces as function spaces on symmetrized bidisc. An analogous decomposition and realization of $\mathbb{A}^2(\mathbb{D}^n)$ under the natural action of the group $\mathfrak{S}_n$ is observed, $\mathfrak{S}_n$ is the permutation group on n symbols. In this talk, we expand our scope to the action of pseudo-reflection groups on the ring of holomorphic functions and in order to do that we obtain an extension of well-known Chevalley-Shephard-Todd Theorem.

Rajeev Gupta, Indian Institute of Technology, Kanpur, India

Title: Weighted Join Operators on Directed Trees

Abstract: A rooted directed tree $\mathscr{T} = (V, E)$ can be extended to a directed graph $\mathscr{T}_\infty = (V_\infty, E_\infty)$ by adding a vertex ∞ to V and declaring each vertex in V as a parent of ∞. Corresponding to $\mathscr{T}_\infty$, a family of semigroup structures $\sqcup_b$ can be associated with extreme ends being induced by the join operation $\sqcup$ and the meet operation $\sqcap$ from lattice theory. Each semigroup structure among these leads to a family of densely defined linear operators W_b^u acting on $\ell^2(V)$, which we refer to as weighted join operators at a given base point $b \in V_\infty$ with prescribed vertex $u \in V$. This class of operators overlaps with the well-studied classes of complex Jordan operators and n-symmetric operators. In this talk we shall study rank one extensions $W_{f,g}$ of weighted join operators W_b^u on rooted directed trees, where $f \in \ell^2(V)$ and $g : V \to \mathbb{C}$ is unspecified. Unlike weighted join operators, these operators are not

necessarily closed. We provide a couple of compatibility conditions involving the weight system and the function g to ensure closedness of $W_{f,g}$.

Hansong Huang, East China University of Science and Technology, China

Title: Boundedness on composition operators between distinct Bergman spaces

Abstract: Composition opertors has been a hot spot at the intersection of operator theory and function theory. Concerning composition operators from one Hilbert reproducing kernel space to another, still there are many issues under consideration. In this talk, we focus on composition operators between distinct Bergman spaces over planar domains. It is shown that the smoothness on boundary of the domain plays an important role in the study. An interplay of function theory, geometry, and operator theory is revealed.

Guoxing Ji, Shaanxi Normal University, China

Title: Subdiagonal algebras with Beurling type invariant subspaces

Abstract: In this talk, we will discuss subdiagonal algebras whose right invariant subspaces in the associated noncommutative H^2 space is of Beurling's type, in a von Neumann algebra. Let $\mathfrak{A}$ be a maximal subdiagonal algebra in a σ-finite von Neumann algebra $\mathcal{M}$. If every right invariant subspace of $\mathfrak{A}$ in the non-commutative Hardy space H^2 is of Beurling type, then we say $\mathfrak{A}$ is of type 1. We determine generators of these algebras and consider a Riesz type factorization theorem for the non-commutative H^1 space. We show that the right analytic Toeplitz algebra on the non-commutative Hardy space H^p associated with a type 1 subdiagonal algebra with multiplicity 1 is hereditary reflexive. Moreover, if a von Neumann algebra is finite, then every subdiagonal algebra of type 1 in it is finite.

Kui Ji, Hebei Normal University, China

Title: Similarity Invariants of Essentially normal Cowen-Douglas Operators and Chern Polynomials

Abstract: In this talk, we will systematically study a class of essentially normal operators by using the geometry method from the Cowen-Douglas theory and prove a Brown-Douglas-Fillmore theorem in the Cowen-Douglas theory. More precisely,

the Chern polynomials and the second fundamental forms are the similarity invariants (in the sense of Herrero) of this class of essentially normal operators.

You Qing Ji and Liu Li, Jilin University, China

Title: The power set of quasinilpotent weighted shifts

Abstract: The power set $\Lambda(V)$ of quasinilpotent operator V was induced by R. G. Douglas and R. Yang in 2016, and it is an impotent indicator of V . In the same paper, the authors showed that V has non-trivial hyperinvariant subspaces if $\Lambda(V))$ is not a singleton. In this talk, we give a class weighted shifts with singleton power set $\{1\}$, and give some weighted shifts with power set $[0, 1]$.

Dinesh Kumar Keshari, National Institute of Science Education Research, Bhubaneswar, India

Title: Similarity invariants of a subclass of the Cowen-Douglas class of operators

Abstract: The explicit description of irreducible homogeneous operators in the Cowen-Douglas class and the localization of Hilbert modules naturally leads to the definition of a smaller class possessing a flag structure. In this talk, I will discuss similarity and unitary invariants of these operators building on my joint work with Kui Ji, Chunlan Jiang, and Gadadhar Misra and more recent joint work with Kui Ji and Chunlan Jiang.

Surjit Kumar, Indian Institute of Science, Bangalore, India

Title: K-homogeneous tuple of operators on bounded symmetric domains

Abstract: Let $\Omega = G/K$ be an irreducible bounded symmetric domain of rank r and dimension d, where K is the maximal compact subgroup of G= Aut(Ω). A commuting d-tuple of operators $T = (T_1, \ldots, T_d)$ is said to be K-homogeneous if for all $k \in K$ the operator tuple $k.T$, which is defined via the usual functional calculus, is unitarily equivalent to T. In this talk, we discuss a classification of K-homogeneous operator tuple under some certain assumptions. Similarity and unitary equivalence of these operator tuples will be also discussed.

This talk is based on joint work with Soumitra Ghara, and Paramita Pramanick.

Shuaibing Luo, Hunan University, China

Title: Hilbert-Schmidtness of some finitely generated submodules in $H^2(\mathbb{D}^2)$

Abstract: A closed subspace M of the Hardy space $H^2(\mathbb{D}^2)$ over the bidisk is called a submodule if it is invariant under multiplication by coordinate functions z_1 and z_2. Whether every finitely generated submodule is Hilbert-Schmidt is an unsolved problem. In this talk we will discuss the Hilbert-Schmidtness of finitely generated submodule M containing $z_1 - z_2$. This is a joint work with Professor Kei Ji Izuchi and Professor Rongwei Yang.

Pan Ma, Central South University, Changsha, China

Title: Big truncated Hankel operators on model spaces

Abstract: For a nonconstant inner function θ and $\varphi \in L^2$, Big Hankel operators H^θ_φ on model spaces $K^2_\theta = H^2 \ominus \theta H^2$ is defined as follows

$$H^\theta_\varphi f = (I - P_\theta)(\varphi f), f \in K^2_\theta$$

where P_θ denotes the orthogonal projection from L^2 onto K^2_θ. In this talk, we will report some progress on boundedness and compactness of big truncated Hankel operators on model spaces. This is based on the joint work with Fugang Yan(Chongqing University) and Dechao Zheng (Vanderbilt University).

Gadadhar Misra, Indian Institute of Science, Bangalore, India

Title: Berger-Shaw theorem for commuting tuple of operators

Abstract: The Berger-Shaw theorem for a single operator states that "a hyponormal operator which is finitely polynomially cyclic has a trace-class self commutator". We introduce a notion of determinant operator associated to block operators and show that it is related to the generalized commutator of a commuting tuple introduced earlier by Helton and Howe. We show that if the determinant operator (or generalized commutator) of a commuting tuple of operators is positive and it is finitely polynomially cyclic, then imposing mild growth conditions, we show that the determinant operator (or generalized commutator) is in trace-class, extending the Berger-Shaw theorem for single operator.

This talk is based on joint work with K. B. Sinha and P. Pramanick.

Avijit Pal, Indian Institute of Technology, Bhillai, India

Title: On Γ_n-contractions and their Conditional Dilations

Abstract: We prove some estimates for elementary symmetric polynomials on $\mathbb{D}^n$. We show that these estimates are sharp which allow us to study the properties of closed symmetrized polydisc Γ_n. Furthermore, we show the existence and uniqueness of solutions to the operator equations

$$S_i - S_{n-i}^* S_n = D_{S_n} X_i D_{S_n} \text{ and } S_{n-i} - S_i^* S_n = D_{S_n} X_{n-i} D_{S_n},$$

where $X_i, X_{n-i} \in \mathcal{B}(\mathcal{D}_{S_n}),$ for all $i = 1, \ldots, (n-1)$, with numerical radius not greater than 1, for a Γ_n-contraction $(S_1, \ldots, S_n)$. We construct a conditional dilation of various classes of Γ_n-contractions. Various properties of a Γ_n-contraction and its explicit dilation allow us to construct a concrete functional model for a Γ_n-contraction. We describe the structure and additional characterization of Γ_n-unitaries and Γ_n-isometries in detail.

Paramita Pramanick, Indian Institute of Science, Bangalore, India

Title: On the Douglas-Yan generalization of the Berger-Shaw theorem

Abstract: A generalization of the Berger-Shaw theorem for commuting tuple $\boldsymbol{T} = (T_1, \ldots, T_n)$ of jointly hyponormal operators was obtained by Douglas and Yan decades ago. In this talk we discuss several examples of this generalization in an attempt to understand if the crucial hypothesis in their theorem requiring the Krull dimension of the Hilbert module over the polynomial ring defined by the map $p \mapsto p(\boldsymbol{T})$ is optimal or not. We will also explore connections with the Arveson-Douglas conjecture of our findings.

This talk is based on joint work with K. B. Sinha and G. Misra

Md. Ramiz Reza, Indian Institute of Technology, Kanpur, India

Title: Wold-type Decomposition and Wandering Subspace Property for a Class of expansive 3 concave operators

Abstract: We discuss the Wold-type decomposition for a class of expansive 3-concave operators. In 2001, Shimorin had shown that if an expansive operator T in $\mathcal{B}(H)$ satisfies the inequality

$$T^{*2}T^2 - 3T^*T + 3I - L^*L - P_{\ker(T^*)} \leq 0,$$

where $L = (T^*T)^{-1}T^*$ is the Moore-Penrose inverse of T and $P_{\ker(T^*)}$ denotes the orthogonal Projection of H onto $\ker(T^*)$, then T is a 3-concave operator which admits Wold type decomposition. We extend this result for a larger class of expansive 3-concave operators. We also study the 3 isometry operators lying in this larger class of expansive 3 concave operators and we find a model of such 3 isometry in terms of Diriclet type Spaces.

This talk is based on joint with Soumitra Ghara and Rajeev Gupta.

Jaydeb Sarkar, Indian Statistical Institute, Bangalore, India

Title: von Neumanns inequality - A brief survey

Abstract: The celebrated von Neumann inequality states that: If T is a linear operator on a Hilbert space H of norm one or less (that is, T is a contraction), then for any polynomial p in C[z], the operator norm of p(T) is dominated by the sup norm of p over the unit disc. The two variables von Neumann inequality (due to Ando) also yields a similar result. These are the consequences of the existence of isometric dilations of single and commuting pairs of contractions (by Sz.-Nagy and Foias and Ando). However, in general, neither the existence of isometric dilation nor the von Neumann inequality holds for n-tuples, n ¿2, of commuting contractions on Hilbert spaces (including finite dimensional Hilbert spaces). In this talk we will provide a taste of isometric dilations, von Neumann inequality and a refined version of von Neumann inequality for n-tuples of commuting contractions. We will also illustrate the success and failure of several variables von Neumann inequality via supporting examples.

Samrat Sen, Indian Institute of Science, Bangalore, India

Title: Geometric invariants for a class of submodules of analytic Hilbert modules

Abstract: Any Hilbert module $\mathcal{H} \subseteq \mathcal{O}(\Omega)$ defines a subsheaf $\mathcal{S}^{\mathcal{H}}$ of the sheaf of holomorphic functions defined on Ω:

$$\mathcal{S}^{\mathcal{H}}(U) := \Big\{ \sum_{i=1}^{n} (f_i|_U) h_i : f_i \in \mathcal{H}, h_i \in \mathcal{O}(U), n \in \mathbb{N} \Big\}, \ U \subseteq \Omega,$$

which is coherent. If the zero set $V_{[\mathcal{I}]}$ of $[\mathcal{I}]$ is a submanifold of codimension t, the stalks of the sheaf $\mathcal{S}^{[\mathcal{I}]}$ are uniformly generated over $V_{[\mathcal{I}]}$. As a result, the following decomposition theorem is obtained for the kernel $K_{[\mathcal{I}]}$ along the zero set $V_{[\mathcal{I}]}$ which generalizes an earlier work by Biswas, Misra and Putinar: There exists

anti-holomorphic maps $F_1, \ldots, F_t : V_{[\mathcal{I}]} \to [\mathcal{I}]$ such that

$$K_{[\mathcal{I}]}(\cdot, u) = \overline{p_1(u)} F_w^1(u) + \cdots \overline{p_t(u)} F_w^t(u),\ u \in \Omega_w,$$

in some neighbourhood Ω_w of each fixed but arbitrary $w \in V_{[\mathcal{I}]}$ for some anti-holomorphic maps $F_w^1, \ldots, F_w^t : \Omega_w \to [\mathcal{I}]$ extending $F_1, \ldots, F_t$.

Shailesh Trivedi, Indian Institute of Technology Kanpur, India

Title: Analytic m-isometries without the wandering subspace property

Abstract: The wandering subspace problem for an analytic norm-increasing m-isometry T on a Hilbert space $\mathcal{H}$ asks whether every T-invariant subspace of $\mathcal{H}$ can be generated by a wandering subspace. An affirmative solution to this problem for $m = 1$ is ascribed to Beurling-Lax-Halmos, while that for $m = 2$ is due to Richter. We capitalize on the idea of weighted shift on one-circuit directed graph to construct a family of analytic cyclic 3-isometries, which do not admit the wandering subspace property and which are norm-increasing on the orthogonal complement of a one-dimensional space. Further, on this one dimensional space, their norms can be made arbitrarily close to 1. We also show that if the wandering subspace property fails for an analytic norm-increasing m-isometry, then it fails miserably in the sense that the smallest T-invariant subspace generated by the wandering subspace is of infinite codimension.

This talk is based on joint work with Akash Anand and Sameer Chavan.

Harald Upmeier, University of Marburg, Germany

Title: Vector-valued Toeplitz Quantization and Discrete Series Representations

Abstract: Generalizing the well-known Toeplitz-Berezin quantization on weighted Bergman spaces over bounded symmetric domains, it is important to study Hilbert spaces of vector-valued holomorphic functions (holomorphic discrete series) and, even more generally, Dolbeault cohomology classes of differential forms (non-holomorphic discrete series). In this generality, even the basic notions such as reproducing kernels, unitary group action and Berezin transform require a careful description. In the talk we present examples of vector-valued Bergman type spaces and derive the Peter-Weyl asymptotic expansion of the matrix-valued Berezin transform in terms of invariant differential operators (joint work with M. Englis). The structure of Toeplitz C^*-algebras in this vector-valued setting depends on a close relationship to non-commutative C^*-duality (crossed product C^*-algebras).

Kai Wang, Fudan University, China

Title: Rigidity of the determinantal point processes

Abstract: In sharp contrast to the classical Bergman kernels, we show the existence of DPP's associated with infinite rank lacunary Bergman kernels which satisfy the Ghosh-Peres number rigidity. This is a joint work with Prof. Yanqi Qiu.

Penghui Wang, Shandong University, China

Title: Essential normality of quotient modules over the polydiscs

Abstract: In this talk, I will talk about the recent development on essential normality of quotient modules of analytic Hilbert modules over the polydiscs. We will give a complete characterization of the essential normality for homogenuous quotient module, which can be seen as the polydisc version of Arveson-Douglas conjection. The talk is based on a series joint work with Kunyu Guo and Chong Zhao.

Yi Wang, State University of New York at Buffalo, USA

Title: A Sharp Inequality of Hardy-Littlewood Type Via Derivatives

Abstract: In this talk, I will introduce a sharp inequality relating a parameterized set of weighted Bergman norms and the Hardy norm on the unit disk. The original form of this inequality can be traced back to 1921, when Carleman provided a complex analytic proof of the famous isoperimetric theorem. In recently years, the inequality has regained attention because of its application in number theory. By taking a close examination of the derivatives of the norms with respect to the parameter, we obtain some sufficient conditions for the inequality to hold. This is joint work with Hui Dan and Kunyu Guo.

Rongwei Yang, SUNY at Albany, USA and Tianjing Normal University, China

Title: A two variable view on the invariant subspace problem

Abstract: By functional operator model theory, every bounded linear operator on a separable Hilbert space can be represented as a conpression of the unilateral shift on the Hardy space over the bidisc $H^2(\mathbb{D}^2)$. Using the reduction operator from

$H^2(\mathbb{D}^2)$ to $H^2(\mathbb{D})$, one can gain a two variable view on the invariant subspace problem for bounded linear operators. This view helps to explain why the problem is difficult.

Yixin Yang, Dalian University of Technology, China

Title: The reducibility and spectrum of compressed shifts on Beurling quotient modules of $H^2(\mathbb{D}^2)$

Abstract: By Sz.-Nagy-Foias model theory, for a scalar-valued inner function u, the compressed shift operator $S(u)$ on the model space $\mathcal{K}_u^2 = H^2(\mathbb{D}) \ominus uH^2(\mathbb{D})$ is unitarily equivalent to a completely nonunitary contraction (c.n.u) with defects $(1, 1)$. $S(u)$ have played an important role in developing both function and operator theory over the past century. In particular, the lattice of invariant subspaces and the spectrum of $S(u)$ have been very well understood. This success led people to study the Hardy space over the bidisk $H^2(\mathbb{D}^2)$. We are interested in the independent behavior of compressed shift operator S_{z_1} or S_{z_2} on the Beurling quotient modules $\mathcal{K}_\theta = H^2(\mathbb{D}^2) \ominus \theta H^2(\mathbb{D}^2)$, where θ is an inner function in two variables z_1, z_2. In this talk, we study the reducibility and spectrum of S_{z_1} for a class of $\mathcal{K}_\theta$. This is joint work with Yufeng Lu and Senhua Zhu.

Dechao Zheng, Vanderbilt University, USA

Title: Compact perturbations of Hankel operators

Abstract: In my talk I will discuss the problem when the product of two Hankel operators is a compact perturbation of a Hankel operator on the Hardy space based on the joint work with Xiaoman Chen, Kunyu Guo and Keiji Izuchi fifteen years ago.

Kehe Zhu, SUNY at Albany, USA, and Shantou University, China
Do Duc Thai **Title:** Sarason's Ha-plitz product problems

Abstract: Sarason's Toeplitz product problem asks for a characterization of analytic functions f and g such that the product $T_f T_{\overline{g}}$ is bounded. Initially, Sarason asked the question in the setting of Hardy and Bergman spaces of the unit disk and a conjecture was also offered. It turned out that Sarason's original conjecture was false for the Hardy and Bergman spaces, but in 2004 it was shown by Cho-Park-Zhu

that the Fock space version of Sarason's conjecture was actually true. In this talk I will present the solution to the Fock space version of Sarason's problem and recent results about the companion problems of characterizing analytic functions f and g such that the Hankel product $H_{\overline{f}}^* H_{\overline{g}}$ or the mixed product $H_{\overline{f}}^* T_{\overline{g}}$ is bounded on the Fock space. Again, there is great divergence between the Hardy/Bergman space setting and the Fock space setting. This is joint work with Cho, Ma, Park, Yan, and Zheng.