

# IWONT 2016

## The 7th International Workshop on Optimal Network Topologies



Tsinghua Sanya International Mathematics Forum  
Sanya, China  
July 11-15, 2016

## Welcome

Welcome to the 7<sup>th</sup> International Workshop on Optimal Network Topologies (IWONT 2016). The first six editions of this workshop took place in Australia, Czech Republic, Spain, Belgium, Indonesia and Slovakia respectively, with a focus on optimal network structures and related topics, including the degree-diameter problem, network design, connectivity, cages, eigenvalue techniques, etc.

The organizing committee for IWONT 2016 consists of Sanming Zhou, Yan-Quan Feng, Joe Ryan, Mei Lu, Rong-Xia Hao and Charl Ras, and the scientific committee consists Mirka Miller (deceased), Jozef Siran, Oriol Serra, Edy Tri Baskoro and Sanming Zhou.

IWONT was started up by Mirka Miller and Brendan McKay in 2005. It is very sad that Mirka is no longer with us. To commemorate her life and many contributions including initiating IWONT, we have organized a Mirka Miller Memorial Talk, which will be given by Rinovia Simanjuntak, a former PhD student of Mirka.

A special issue for IWONT 2016 will be published in *J. Interconnection Networks*. If you would like to submit a paper to this special issue, please send your submission in PDF format to [sanming@unimelb.edu.au] by September 30, 2016.

Following a tradition of IWONT, participation of this workshop is mostly by invitation. The organizing committee would like to thank you the participants for accepting our invitation and delivering talks.

We hope your time in Sanya is productive and enjoyable.

Sanming Zhou  
Chair of the Organizing Committee, IWONT 2016

## **Information**

### **Workshop Venue**

All talks will take place in Conference Room No. 1, Building B of TSIMF, except two parallel sessions on Tuesday afternoon which will be in Conference Room No. 2 in the same building.

### **Meals**

All meals (buffet except for the conference dinner) beginning with the dinner of Sunday 10 July and ending at the dinner of Friday 15 July will be provided and will be in the Dining Hall, Building B of TSIMF. Soft drinks will be provided in each meal, and alcoholic drinks can be purchased at your own expense in a store next to the restaurant. In general, there will have some vegetarian food in each meal. However, please advise TSIMF if you are a vegetarian or you have any dietary restriction.

Meal times at TSIMF are as follows:

Breakfast: 7:30-8:30

Lunch: 12:00-13:30

Dinner: 18:00-19:30

### **Tea Breaks**

Morning tea: 10:00-10:30

Afternoon tea: 15:00-15:30

### **Workshop Dinner**

There will have a workshop dinner at 18:00pm on Thursday 14 July at Dining Hall, Building B of TSIMF.

### **Workshop Photos**

Workshop photos will be taken during 12:00pm-12:15pm on Tuesday 12 July.

### **Wireless Network Access**

Wireless network access in both of the conference rooms and guest rooms will be provided and no password is needed.

### **Sports Facilities of TSIMF**

Sports facilities of TSIMF include a swimming pool and two tennis courts. It would be a good idea to bring your swimming gear with you.

### **Transportation between the airport and TSIMF**

TSIMF will organize delivery from the airport to TSIMF on your arrival and to the airport when you leave TSIMF. In case you have not done it, please provide your arrival and departure information to TSIMF to make sure you will be booked for this service.

## **Local Transportation**

Buses between TSIMF and Sanya Seaside:

Buses between TSIMF (near the entrance of TSIMF) and Sanya Seaside (Haipo Square (海坡广场) of Howard Johnson) will be operated from July 11 to July 15, 2016 at the following time schedule:

09:00	TSIMF to Haipo Square (海坡广场)
11:00	Haipo Square (海坡广场) to TSIMF
18:30	TSIMF to Haipo Square (海坡广场)
21:00	Haipo Square (海坡广场) to TSIMF

Please be on time and catch bus at right place in order to avoid disappointment.

Taxi and shuttle bus:

TSIMF can help arrange taxi or shuttle bus on your own cost. Please contact our staff at the reception desk of TSIMF if you need a taxi or shuttle bus.

## **Wednesday Afternoon**

TSIMF will provide free transportation to Yalong Bay Tropical Paradise Forest Park (亚龙湾热带天堂森林公园) on Wednesday 13 July afternoon, departing at 13:00pm from TSIMF and returning from Yalong Bay at 17:30pm. However, participants are required to purchase entrance tickets by themselves in tourist areas by cash (Chinese currency).

## **July 16<sup>th</sup>**

Participants who plan to depart on July 16<sup>th</sup> are welcome to stay in TSIMF on the night of July 15<sup>th</sup>. TSIMF will be able to cover the accommodation on the night of July 15<sup>th</sup>, but unfortunately participants will need to pay the expenses for meals and transportation. Formal receipts can be issued for the expenses for meals but not for transportation.

The standard charge for lifting to the airport is: RMB 15 for one trip in a four seats car, and RMB 30 for one trip in a six seats commercial vehicle.

## **Acknowledgements**

The organizing committee gratefully acknowledges the generous support of Tsinghua Sanya International Mathematics Forum. Special thanks go to Ms. Yanyu Fang at TSIMF for efficiently handling most administrative issues.

## Monday

08:50 - 09:00	Opening
09:00 - 10:00	Plenary: <i>Jozef Širáň</i>
10:00 - 10:30	Morning tea
	<b>Morning session (Room No. 1)</b>
10:30 - 11:00	Martin Bachratý
11:00 - 11:30	Camino Balbuena
11:30 - 12:00	Novi Herawati Bong
12:00 - 13:30	Lunch
	<b>Early-afternoon session (Room No. 1)</b>
13:30 - 14:00	Dominique Buset
14:00 - 14:30	Francesc Comellas
14:30 - 15:00	Robert R. Lewis
15:00 - 15:30	Afternoon tea
	<b>Late-afternoon session (Room No. 1)</b>
15:30 - 16:00	Grahame Erskine
16:00 - 16:30	Yan-Quan Feng
16:30 - 17:00	Rong-Xia Hao

## Tuesday

09:00 - 10:00	Plenary: <i>Genghua Fan</i>	
10:00 - 10:30	Morning tea	
	<b>Morning session (Room No. 1)</b>	
10:30 - 11:00	Sakander Hayat	
11:00 - 11:30	Martin Mačaj	
11:30 - 12:00	James Tuite	
12:00 - 12:15	Photos	
12:00 - 13:30	Lunch	
	<b>Early-afternoon session 1 (Room No. 1)</b>	<b>Early-afternoon session 2 (Room No. 2)</b>
13:30 - 14:00	Leif K. Jørgensen	Sun-Yuan Hsieh
14:00 - 14:30	Nacho López Lorenzo	Charl Ras
14:30 - 15:00	Diego A. G. Moreno	Patrick Andersen
15:00 - 15:30	Afternoon tea	
	<b>Late-afternoon session 1 (Room No. 1)</b>	<b>Late-afternoon session 2 (Room No. 2)</b>
15:30 - 16:00	Liyang Kang	Martin Knor
16:00 - 16:30	Yuqing Lin	Jana Šiagiová
16:30 - 17:00	Lili Hu / Xiangwen Li	Jin-Xin Zhou

## Wednesday

09:00 - 10:00	Plenary: <i>Miquel Angel Fiol</i>
10:00 - 10:30	Morning tea
10:30 - 11:30	Mirka Miller memorial talk: <i>Rinovia Simanjuntak</i>
11:30 - 12:00	Shenggui Zhang
12:00 - 13:00	Lunch
13:00 -	Free afternoon

## Thursday

09:00 - 10:00	Plenary: <i>Xuding Zhu</i>
10:00 - 10:30	Morning tea
	<b>Morning session (Room No. 1)</b>
10:30 - 11:00	Christina Dalfo
11:00 - 11:30	Miret Biosca Josep Maria
11:30 - 12:00	Xiaogang Liu
12:00 - 13:30	Lunch
	<b>Early-afternoon session (Room No. 1)</b>
13:30 - 14:00	Felix Lazebnik
14:00 - 14:30	Li-Da Tong
14:30 - 15:00	Guillermo Pineda-Villavicencio
15:00 - 15:30	Afternoon tea
	<b>Late-afternoon session (Room No. 1)</b>
15:30 - 16:00	Jinjiang Yuan
16:00 - 16:30	Cunquan Zhang
16:30 - 17:00	Xiao-Dong Zhang
18:00 - 20:00	Conference dinner

## Friday

09:00 - 10:00	Plenary: <i>Jack H. Koolen</i>
10:00 - 10:30	Morning tea
	<b>Morning session (Room No. 1)</b>
10:30 - 11:00	Muhammad Adib Surani
11:00 - 11:30	Sandi Klavžar
11:30 - 12:00	Gabriela Araujo-Pardo
12:00 - 13:30	Lunch

# Plenary Talks

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1. Genghua Fan, Center for Discrete Mathematics Fuzhou University, China

**Title:** Large Even Factors of Graphs

**Abstract:** A spanning subgraph  $F$  of a graph  $G$  is called an even factor of  $G$  if each vertex of  $F$  has even degree at least 2 in  $F$ . It was proved that if a graph  $G$  has an even factor, then it has an even factor  $F$  with  $E(F) \geq \frac{2}{7}(|E(G)| + 1)$ , which is best possible. Recently, Cheng et al. extended the result by considering vertices of degree 2. They proved that if a graph  $G$  has an even factor, then it has an even factor  $F$  with  $E(F) \geq \frac{2}{7}(|E(G)| + 1) + \frac{2}{7}|V_2(G)|$ , where  $V_2(G)$  is the set of vertices of degree 2 in  $G$ . They also gave examples showing that the second coefficient cannot be larger than  $\frac{2}{7}$  and conjectured that if a graph  $G$  has an even factor, then it has an even factor  $F$  with  $E(F) \geq \frac{2}{7}(|E(G)| + 1) + \frac{2}{7}|V_2(G)|$ . We note that the conjecture is false if  $G$  is a triangle. We confirm the conjecture for all graphs on at least 4 vertices. Moreover, if  $E(F) \leq \frac{2}{7}(|E(G)| + 1) + \frac{2}{7}|V_2(G)|$ , for every even factor  $H$  of  $G$ , then every maximum even factor of  $G$  is a 2-factor in which each component is an even circuit.

2. Miquel Angel Fiol, Universitat Politècnica de Catalunya

**Title:** Equivalent Characterizations of the Spectra of Graphs and Some Applications

**Abstract:** As it is well-known, the spectrum  $spG$  (of the adjacency matrix  $A$ ) of a graph  $G$ , with  $d$  distinct eigenvalues other than its spectral radius  $\lambda_0$ , usually provides a lot of information about the structure of  $G$ . Moreover, from  $spG$  we can define the so-called predistance polynomials  $p_0, \dots, p_d \in \mathbb{R}_d[x]$ , with  $dgr p_i = i$ , for  $i = 0, \dots, d$ , which are orthogonal with respect to the scalar product  $\langle f, g \rangle_G = \frac{1}{n}tr(f(A)g(A))$  and normalized in such a way that  $\|p_i\|_G^2 = p_i(\lambda_0)$ . They can be seen as a generalization for any graph of the distance polynomials of a distance-regular graph. Going further, we consider the preintersection numbers  $\xi_{ij}^h$  for  $i, j, h \in \{0, \dots, d\}$ , which generalize the intersection numbers of a distance-regular graph, and they are the Fourier coefficients of  $p_i p_j$  in terms of the basis  $\{p_h\}_{0 \leq h \leq d}$ . The aim of this talk is to show that, for any graph  $G$ , the information contained in its spectrum, preintersection polynomials, and preintersection numbers is equivalent. Also, we give some characterizations of distance-regularity which are based on the above concepts. For instance, we comment upon the so-called spectral excess theorem stating that a connected regular graph  $G$  is distance-regular if and only if its spectral excess, which is the value of  $p_d$  at  $\lambda_0$ , equals the average excess, that is, the mean of the numbers of vertices at extremal distance  $d$  from every vertex. (This is joint work with V. Diego and J. Fàbrega.)

3. Jack H. Koolen, School of Mathematical Sciences, University of Science and Technology of China

**Title:** Graphs with Two Main and Two Plain Eigenvalues

**Abstract:** In 1970 Doob asked to study graphs with a small number of distinct eigenvalues. In this talk, we study graphs with exactly two main and two plain distinct eigenvalues. This is joint work with S. Hayat and M. Javaid.

4. Jozef Širáň, Open University and Slovak University of Technology

**Title:** The Degree-diameter Problem for Vertex-transitive and Cayley Graphs

**Abstract:** Along with the classical degree-diameter problem for graphs there has been growing interest in a number of versions in which the family of graphs under consideration is restricted. An important and fruitful restriction appears to be symmetry, linking the problem with interesting questions in group theory. We will thus focus on the vertex-transitive and Cayley version of the degree-diameter problem, that is, on determining, or at least estimating, the largest order of a vertex-transitive and a Cayley graph of a given degree and diameter. We will survey the available results on constructions of ‘large’ vertex-transitive and Cayley graphs of given degree and diameter and on attempts to improve the Moore bound for graphs with a high ‘level of symmetry’.

5. Xuding Zhu, Zhejiang Normal University, China

**Title:** Twisted Hypercubes

**Abstract:** The family of twisted hypercubes are defined as follows: There is one 0-dimension twisted hypercube, consisting of a single vertex, i.e.,  $K_1$ . For  $n \geq 1$ , an  $n$ -dimension twisted hypercube is obtained from two  $(n - 1)$ -dimension twisted hypercubes, say  $H$  and  $H'$ , by adding a perfect matching connecting each vertex of  $H$  with one vertex of  $H'$ . Twisted hypercubes is a generalization of hypercubes. A hypercube is a twisted hypercube, but the family of twisted hypercubes is a much larger family of graphs. Each  $n$ -dimensional twisted hypercube is an  $n$ -regular graph with  $2^n$  vertices. The  $n$ -dimension hypercube has diameter  $n$ . However, general  $n$ -dimension twisted hypercubes usually have smaller diameter. First we present an explicit construction of a family of twisted hypercubes  $H_n : n = 1, 2, \dots$ , where  $H_n$  is an  $n$ -dimension twisted hypercube, which is obtained from two copies of  $H_{n-1}$  by adding a perfect matching. We prove that  $H_n$  has diameter  $(1+o(1))n/\log_2 n$ , which is asymptotically optimal, as any  $n$ -regular graph on  $2^n$  vertices has diameter larger than  $n/\log_2 n$ . Next we prove that a random  $n$ -dimension twisted hypercube not only have diameter  $(1 + o(1))n/\log_2 n$ , it almost surely had  $n$ -wide diameter  $(1 + o(1))n/\log_2 n$ , i.e., between any two vertices of a random  $n$ -dimension twisted hypercube, there are  $n$  internally vertex disjoint paths, each having length  $(1 + o(1))n/\log_2 n$ . The later result is a joint work with Dudek, Perez-Gimenez, Pralat, Qi and West.



# Mirka Miller Memorial Talk

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Rinovia Simanjuntak, Institut Teknologi Bandung, Indonesia

**Title:** Mirka, the Degree/Diameter Problem, and Beyond

**Abstract:** The degree/diameter problem is to determine the largest graphs or digraphs of given maximum degree and given diameter. This talk will present an account of some of Mirka Miller's contribution to the degree/diameter problem. I shall also include my own experiences and impressions of Mirka as a source of inspiration in both mathematical research and many other aspects of life.

# Invited Talks

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1. Patrick J. Andersen, The University of Melbourne, Australia

**Title:** Minimum Bottleneck Spanning Trees with Degree Bounds

**Abstract:** Given a graph  $G$  with edge lengths, the minimum bottleneck spanning tree (MBST) problem is to find a spanning tree where length of the longest edge in tree is minimum. It is a well known fact that every minimum spanning tree (MST) is a minimum bottleneck spanning tree. In this talk, I will introduce the  $\delta$ -MBST problem, which is the problem of finding an MBST such that every vertex in the tree has degree at most  $\delta$ . In this case, the optimal solutions to the similarly defined  $\delta$ -MST problem are not necessarily optimal solutions to the  $\delta$ -MBST. This talk will focus on complexity results and approximation algorithms for the  $\delta$ -MBST problem.

2. Martin Bachratý, Department of Mathematics, Faculty of Civil Engineering, Slovak University of Technology, Slovak

**Title:** The Degree-diameter Problem for Cayley Graphs and Finite Geometries

**Abstract:** Let  $n(d; k)$  denote the largest order of a graph of diameter  $k$  and maximum degree  $d$ . The well known Moore bound states that  $n(d; k)$  cannot exceed  $M(d; k) = 1 + \sum_{i=1}^k d(d-1)^{i-1}$ . By a classical result of Delorme, for  $k = 2, 3$  and  $5$  the Moore bound can be approached asymptotically in the sense that

$$\limsup_{d \rightarrow \infty} n(d, k)/M(d, k) = 1.$$

Recent results show that for diameters  $2$  and  $3$  this is true also if we restrict ourselves to Cayley graphs and the corresponding families can be constructed from finite generalized polygons. We briefly introduce this method and present problems that occur when applying this method to the case of diameter  $5$ .

3. Camino Balbuena, Departament d'Enginyeria Civil i Ambiental, Universitat Politècnica de Catalunya, Barcelona, Spain

**Title:** New Small Regular Graphs of Girth  $5$

**Abstract:** A  $(k; g)$ -graph is a  $k$ -regular graph with girth  $g$ , and a  $(k; g)$ -cage is a  $(k; g)$ -graph with the fewest possible number of vertices. The Cage Problem consists on constructing regular graphs of given girth  $g$  and minimum order. We focus on girth  $g = 5$ , where cages are known only for degrees  $k \leq 7$ . We construct  $(k; 5)$ -graphs using techniques exposed by Funk [Note di Matematica. 29 suppl.1, (2009) 91 - 114] and Abreu et al. [Discrete Math. 312 (2012), 2832 - 2842] to obtain the best upper bounds known hitherto. The talk is mainly based on the joint work with E. Abajo, G. Araujo-Pardo and M. Bendala.

4. Novi Herawati Bong, The University of Newcastle, Australia

**Title:** Maximum Degree-Diameter Bounded Subgraph on Hyperdiamond

**Abstract:** Maximum degree and diameter-bounded subgraph (MaxDDBS) problem is a generalisation of the degree-diameter problem (DDP) in which the aim is to maximise the number of vertices in a graph with given degree and diameter. In MaxDDBS, the selection of edges is restricted by the requirement that the resulting graph be a subgraph of some given host architecture/network. The MaxDDBS problem has been studied for rectangular mesh, honeycomb grid, Benes network among others. In this talk we will discuss the results of MaxDDBS problem on honeycomb network and continue on a natural generalisation of honeycomb networks, which is the hyperdiamond. This talk is based on the joint works with Joe Ryan and Yuqing Lin.

5. Dominique Buset, Ecole Polytechnique de Bruxelles, Universit Libre de Bruxelles, Belgium

**Title:** The Degree/Diameter Problem for Mixed Graphs - A New Upper Bound

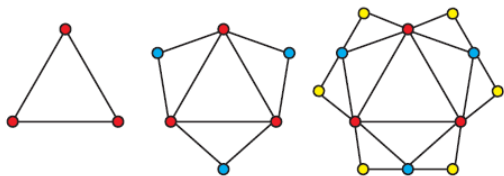
**Abstract:** The Degree/Diameter Problem for graphs has its motivation in the efficient design of interconnection networks. It seeks to find the maximum possible order of a graph with a given (maximum) degree and diameter. It is known that graphs attaining the maximum possible value (the Moore bound) are extremely rare, but much activity is focused on finding new examples of graphs or families of graph with orders approaching the bound as closely as possible. This problem was first mention in 1964 and has its motivation in the efficient design of interconnection networks. A lot of great mathematician studied this problem and obtained some results but there still remain a lot of unsolved problems about this subject. Our regretted professor Mirka Miller has given a great expansion to this problems and a lot of new results were given by her and her students. One of the problem she was recently interested in, was the Degree/Diameter problem for mixed graphs i.e. graphs in which we allow undirected edges and arcs (directed edges). In this talk, well provide a new upper bound for the maximal number of vertices of a mixed graph with given out-degree and diameter. We also provide some nice properties related to the sequence used in the new formula. The talk is mainly based on the joint work with M. El Amiri, G. Erskine, Mirka Miller and Hebert Perez-Roses.

6. Francesc Comellas, Universitat Politècnica de Catalunya, Barcelona, Catalonia, Spain

**Title:** On the Normalized Laplacian Spectra of Iterated Triangulations of Graphs

**Abstract:** The spectra of different matrices associated to a graph provide information on its diameter, degree distribution, total number of links, paths of a given length, local clustering, community structure, number of spanning trees and many other invariants. In particular, dynamical aspects of a network, such as its synchronizability and random walk properties, can be obtained from the eigenvalues of the Laplacian and normalized Laplacian matrices. From them it is possible to calculate several interesting parameters and invariants which measure the efficiency of navigation on a network, like the hitting and mixing times and Kemeny's constant. In this talk we first recall an operation, called triangulation, that can be applied to any simple connected graph. The triangulation of a graph  $G$ , denoted by  $\tau_m(G)$ , is

the graph obtained by adding  $m$  new vertices to each edge of  $G$  and by joining each of them to the end vertices of their corresponding edge.



*An example of the triangulation operation for  $m = 1$ . Red vertices denote the three initial  $K_3$  vertices, while blue and yellow vertices have been introduced in the first and second triangulation operation,  $\tau(K_3)$  and  $\tau^2(K_3)$ . The resulting graphs are known as recursive clique trees or pseudofractal scale-free web.*

We determine the spectra of the normalized Laplacians of iterated triangulations of any simple connected graph and we discuss their structure. The results are used to calculate three significant graph invariants: the multiplicative degree-Kirchhoff index, Kemeny's constant and the number of spanning trees.

7. Cristina Dalfo, Universitat Politecnica de Catalunya, Barcelona, Spain

**Title:** On the Order of Iterated Line Digraphs

**Abstract:** Given a digraph  $G$ , we propose a new method to find the recurrence equation on the number of vertices  $n_k$  of the  $k$ -iterated line digraph  $L^k(G)$ , for  $k \geq 0$ , where  $L^0(G) = G$ . We obtain this result by using the minimal polynomial of a quotient digraph  $\pi(G)$  of  $G$ . We show some examples of this method applied to the so-called cyclic Kautz, the unicyclic, and the acyclic digraphs. In the first case, our method gives the enumeration of the ternary length-2 squarefree words of any length. This is joint work with M. A. Fiol.

8. Grahame Erskine, Open University, UK

**Title:** Searching for Mixed Moore Cayley Graphs

**Abstract:** In the degree-diameter problem, it is well known that for undirected graphs it is only possible to achieve the largest theoretical order of graph for a given diameter and maximum degree (the Moore bound) in a handful of cases. By contrast, in the mixed case where we allow both directed arcs and undirected edges, there are an infinite number of feasible parameter sets where the existence of a Moore graph with these parameters is unknown. A natural way to attack this problem is to exploit group-theoretic ideas to search for possible Cayley graphs attaining the Moore bound. We discuss some new ideas which allow us to extend the search for such graphs significantly beyond currently reported results.

9. Yan-Quan Feng, Mathematics, Beijing Jiaotong University

**Title:** Fault-free Cycles on Hypercubes and Folded Hypercubes Under the Conditional-fault mode

**Abstract:** It is well-known that the  $n$ -dimensional hypercube  $Q_n$  is one of the most versatile and efficient interconnection network architecture yet discovered for building massively parallel or distributed systems. The  $n$ -dimensional folded hypercube  $FQ_n$  is a variation of the hypercube structure, which is constructed from a hypercube by

adding an edge of every pair of vertices that have complementary addresses. Let  $F$  be the faulty set of  $Q_n$  or  $FQ_n$ , and let  $f_v, f_e$  be the numbers of faulty vertices and faulty edges in  $F$ , respectively. An edge  $e = (x; y)$  is said to be free if  $e, x, y$  are not in  $F$ , and a cycle is said to be fault-free if there is no faulty vertex or faulty edge on the cycle. Assume that each fault-free vertex of  $Q_n$  or  $FQ_n$  is incident to at least two free edges, called the conditional-fault model. Under the conditional-fault model, we review some new results, given by the author and his colleagues, about fault-free cycles on  $Q_n$  and  $FQ_n$  based on  $f_v$  and  $f_e$ , of which one confirms a conjecture proposed by Tsai (2007).

10. Rong-Xia Hao, Department of Mathematics, Beijing Jiaotong University, China

**Title:** Fault Tolerance of the Balanced Hypercube  $BH_n$

**Abstract:** The  $n$ -dimensional balanced hypercube  $BH_n$ , as a variant of the hypercube network, is an important network for parallel processing and computing. Since failure may occur when the network is put into use, this motivates to consider fault tolerance of  $BH_n$ . In this talk, I will give some properties of  $BH_n$  and show some results about fault-tolerant hamiltonicity; edge-hamiltonicity; bipancyclicity and edge-bipancyclicity of  $BH_n$ .

11. Sakander Hayat, School of Mathematical Sciences, University of Science and Technology of China, China

**Title:** Hypercubes are Determined by Their Distance Spectra

**Abstract:** We use the theory of isometric embedding and metric hierarchy of graphs to prove that the hypercubes are determined by their distance spectra. Moreover, we prove some bounds on the distance eigenvalues of graphs. We also study the graphs with exactly three distinct distance eigenvalues. This is a joint work with Jack Koolen and Quaid Iqbal.

12. Sun-Yuan Hsieh, Department of Computer Science and Information Engineering, National Cheng Kung University, Taiwan

**Title:** The Steiner Tree Problem and Its Variants

**Abstract:** Given a graph  $G = (V, E)$  with a cost (or distance) function  $c : E \rightarrow R^+$ , and a vertex subset  $R \subseteq V$ , a *Steiner tree* is a connected and acyclic subgraph of  $G$  that contains all the vertices in  $R$ . The cost of a *Steiner tree* is the sum of the costs of all the edges in the tree. The *Steiner Tree Problem* (STP) involves finding a *Steiner tree* with the minimum cost in  $G$ . In this talk, the classical STP and its variant problems including Euclidean STP, rectilinear STP, internal STP (ISTP), selected internal STP (SISTP), terminal STP (TSTP), and partial-terminal STP (PTSTP) will be introduced. Moreover, an approximation algorithm for PTSTP with ratio 1.67 will be presented.

13. Leif K. Jørgensen, Department of Mathematical Sciences, Aalborg University, Denmark

**Title:** Total Graph Coherent Configurations: New Graphs from Moore Graphs

**Abstract:** For a graph  $\Gamma$ , the total graph  $T(\Gamma)$  has vertex set  $V(\Gamma) \cup E(\Gamma)$  and adjacency in  $T(\Gamma)$  means adjacency/incidence in  $\Gamma$ . The automorphism group  $G$  of  $T(\Gamma)$  is usually isomorphic to the automorphism group of  $\Gamma$ . The edge set of the complete graph with vertex set  $V(T(\Gamma))$  is partitioned in orbits under the action of  $G$ . (Alternatively, we may consider a coarser, combinatorially defined partition, called the coherent configuration generated by  $T(\Gamma)$ .) Our goal is to construct a new graph with vertex set  $V(T(\Gamma))$  and edge set a union of some of the orbits, and with automorphism group larger than  $G$ .

In particular we consider the case when  $\Gamma$  is the complement of a Moore graph. If  $\Gamma$  is the complement of the Petersen graph or the Hoffman-Singleton graph then we get graphs (in fact 4 class association schemes) of order 40 and 1100 with large automorphism groups. We will also discuss some properties for the case when  $\Gamma$  is the complement of a Moore graph of degree 57. This is joint work with M. Klin and M. Ziv-Av, Ben-Gurion University of the Negev.

14. Miret Biosca Josep Maria and Anita A. Sillassen, Universitat de Lleida, Spain, and Aalborg University, Denmark

**Title:** Looking for Digraphs of Excess One

**Abstract:** A  $k$ -geodetic digraph  $G$  is a digraph in which, for every pair of vertices  $u$  and  $v$  (not necessarily distinct), there is at most one walk of length  $\leq k$  from  $u$  to  $v$ . If the diameter of  $G$  is  $k$ , we say that  $G$  is strongly geodetic. The order  $N(d, k)$  of a  $k$ -geodetic digraph of minimum out-degree  $d$  fulfills  $N(d, k) \geq 1 + d + d^2 + \dots + d^k = M(d, k)$ , where  $M(d, k)$  is the Moore bound. This bound is attained if and only if the diameter is  $k$ . Thus, strongly geodetic digraphs only exist for  $d = 1$  or  $k = 1$ . Hence, for  $d, k \geq 2$  we wish to determine if  $N(d, k) = M(d, k) + 1$ . A  $k$ -geodetic digraph with minimum out-degree  $d$  and order  $M(d, k) + 1$  is denoted as a  $(d, k, 1)$ -digraph or said to have excess 1.

In this talk we will show that  $(d, k, 1)$ -digraphs are diregular. Thus digraphs with excess one do not exist for  $d = 2$ . We will also study the equation  $I + A + \dots + A^k = J - P$  fulfilled by the adjacency matrix  $A$  of such a digraph, where  $J$  denotes the all-one matrix and  $P$  is a *permutation matrix*. Since the eigenvalues of  $P$  are roots of unity, the factorization in  $\mathbb{Q}[x]$  of the characteristic polynomial of  $A$  involves the polynomials  $F_{n,k}(x) = \Phi_n(1 + x + \dots + x^k)$ , where  $\Phi_n(x)$  denotes the  $n$ th cyclotomic polynomial. These polynomials also appear in the factorization in of the characteristic polynomial of digraphs with defect one. We will prove, by using algebraic techniques, the nonexistence of  $(d, k, 1)$ -digraphs for  $k = 2$  when  $d > 7$  and for  $k = 3, 4$  when  $d > 1$ .

This is a joint work with Mirka Miller.

15. Liying Kang, Department of Mathematics, Shanghai University, China

**Title:** Solution to a Forcible Version of a Graphic Sequence Problem

**Abstract:** Let  $A_n = (a_1; a_2; \dots; a_n)$  and  $B_n = (b_1; b_2; \dots; b_n)$  be nonnegative integer sequences with  $A_n \leq B_n$ . The sequence of this note is to give a good characterization

such that every integer sequence  $\pi = (d_1; d_2; \dots; d_n)$  with even sum and  $A_n \leq \pi \leq B_n$  is graphic. This solves a forcible version of problem posed by Niessen and generalizes the Erdős-Gallai theorem.

16. Sandi Klavžar, University of Ljubljana, University of Maribor and Institute of Mathematics, Physics and Mechanics, Slovenia

**Title:** Weak Edge Geodetic Problem

**Abstract:** If  $G = (V(G); E(G))$  is a graph, then a set  $S \subseteq V(G)$  is called a weak edge geodetic cover if for any pair  $x; y \in S$  there exists a shortest  $x; y$ -path  $P_{xy}$  such that  $\bigcup_{x,y \in S} E(P_{xy}) = E(G)$ . A weak edge geodetic cover of minimum cardinality is a weak edge geodetic basis, its cardinality is the weak edge geodetic dimension  $wge(G)$  of  $G$ . The weak edge geodetic problem is to find a weak edge geodetic basis of  $G$ . In this talk it will be first shown that the weak edge geodetic problem is NP-complete. Afterwards several sharp bounds for the weak edge geodetic dimension will be presented as well as exact or nearly exact values for different types of networks. The talk is mainly based on the joint work with Paul Manuel, \*\*\*\*\*

17. Martin Knor, Slovak University of Technology in Bratislava, Bratislava, Slovakia

**Title:** Graphs with the minimal Balaban Index

**Abstract:** We study graphs of order  $n$  with the minimal value of Balaban index. The Balaban index is defined as

$$J(G) = \frac{m}{m - n + 2} \sum_{uv \in E(G)} \frac{1}{\sqrt{w(u) \cdot w(v)}},$$

where the sum is taken over all edges of  $G$  and for  $x \in V(G)$  we have  $w(x) = \sum_{y \in V(G)} \text{dist}(x, y)$ .

We show that  $J(G) \geq 4/(n - 1)$  and when  $n$  is large then  $J(G) > 8/n + o(n^{-1})$ . Then, for small values of  $n$  we determine the extremal graphs. Finally, we show that balanced dumbbell graphs with clique size  $\sqrt[4]{\pi/2}\sqrt{n} + o(\sqrt{n})$  have the value of Balaban index about  $10.15/n$ . Finally, we present a conjecture about the structure of extremal graphs.

18. Lili Hu and Xiangwen Li, Huazhong Normal University, Wuhan, China

**Title:** On 3-Coloring of Signed Planar Graphs

**Abstract:** In this paper, we first prove the signed graph version of Grötzsch's 3-coloring theorem: every triangle-free signed planar graph is 3-colorable. We then investigate the signed graph version of Erdős problem: Is there a constant  $c$  such that every planar signed graph without  $k$ -cycle, where  $4 \leq k \leq c$ , is 3-colorable and prove that each signed planar graph without cycles of length from 4 to 8 is 3-colorable.

19. Felix Lazebnik, Department of Mathematical Sciences, University of Delaware, USA

**Title:** On Some Families of Algebraically Defined Graphs and Digraphs

**Abstract:** In this talk I will present some recent results, open problems and conjectures related to the structure of graphs and digraphs defined by certain systems of algebraic equations. I will also mention several applications of these constructions.

20. Robert R. Lewis, The Open University, UK. USA

**Title:** The Degree-diameter Problem for Circulant Graphs of Degree 10 and 11

**Abstract:** For degrees 10 and 11, newly-discovered families of undirected circulant graphs of arbitrary diameter are presented which are largest known and are conjectured to be extremal. The orders and generating sets of these graphs are defined as polynomial functions of the diameter. These graphs are consistent with an earlier conjecture about the order of extremal circulant graphs of any degree and diameter.

21. Yuqing Lin, School of Electrical Engineering and Computer Science, The University of Newcastle, Australia

**Title:** Factorization of Regular Graphs

**Abstract:** A  $1$ -factorization partitions the edges of a graph into disjoint  $1$ -factors. A graph  $G$  is said to be  $1$ -factorable if it admits a  $1$ -factorization. Let  $G$  be a  $k$ -regular graph with  $2n$  nodes. If  $k$  is sufficiently large, then  $G$  has to be  $1$ -factorable. For example, it is easy to see this is true for trivial cases such as  $k = 2n - 1$  or if  $k = 2n - 2$ . The  $1$ -factorization conjecture is a long-standing conjecture that states that  $k \approx n$  is sufficient. i.e.,  $k = n - 1$  then  $G$  is  $1$ -factorable. In 1985, Chetwynd and Hilton [1] show that if  $k = 12n/7$ , then  $G$  is  $1$ -factorable. And Hilton [3] proved that  $G$  contains at least  $\lfloor k/3 \rfloor$  edge-disjoint  $1$ -factors if  $k = n$ . Zhang and Zhu [2] has improved that result by showing that  $G$  contains at least  $\lfloor k/2 \rfloor$  edge disjoint  $1$ -factors. We have been making progress along this direction until recently, in [4], Casaba *et al.* has completely solved the  $1$ -factoration conjecture using probabilistic approach, they have shown that if  $k \geq 2\lfloor n/2 \rfloor - 1$ , then every  $k$ -regular graph  $G$  on  $2n$  vertices has a decomposition into perfect matchings, i.e. the graph  $G$  is  $1$ -factorable.

However, following the same approaches we have been using, we have obtained some new results. Previous work focus on graphs with degree more than half amount of the vertices, we have looked at the graphs with smaller degree and obtained the following results

**Theorem 1.** *Let  $k, n$  be two integers such that  $k = \lfloor (n - 1)/2 \rfloor$ . Let  $G$  be  $k$ -regular graphs with order  $n$ . If  $G$  is  $k$ -edge-connected, then  $G$  contains  $k$  edge-disjoint perfect matchings.*

In this talk, I will briefly discuss this result and the techniques we used.

## REFERENCES

- [1] A. G. Chetwynd, A. J. W. Hilton, Regular graphs of high degree are  $1$ -factorizable, Proceedings of the London Mathematical Society **50** (2): 193-206,
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- [4] B. Csaba, D. Kühn, A. Lo, D. Osthus, A. Treglown, Proof of the 1-factorization and Hamilton Decomposition Conjectures. To appear in the *Memoirs of the AMS*.

22. Xiaogang Liu, Department of Applied Mathematics, Northwestern Polytechnical University, China

**Title:** Constructing Expander Families From Known Ones

**Abstract:** Let  $\{G_i\}_{i \geq 1}$  be an infinite sequence of  $r$ -regular graphs such that  $|V(G_i)| \rightarrow \infty$  as  $i \rightarrow \infty$ . Let  $\lambda_2(G_i)$  be the second largest eigenvalue of the adjacency matrix of  $G_i$ . If there exists a real number  $\epsilon > 0$  such that  $r - \lambda_2(G_i) \geq \epsilon$  for all  $i$ , then  $\{G_i\}_{i \geq 1}$  is called an  $\epsilon$ -expander family.

Graph operations are natural techniques for producing new graphs from old ones. In this talk, I will introduce some basic results on constructing expander families from known ones by using graph operations. I will also present some results we have obtained and mention some future research works.

23. Nacho López Lorenzo, Departament de Matemàtica, Universitat de Lleida

**Title:** On Mixed Graphs of Diameter Two and Order Close to the Moore Bound.

**Abstract:** Network topologies based on mixed graphs arise in many practical situations, where the relationship between nodes in the network can be undirected or directed depending on whether the communication between nodes is two-way or only one-way. From this point of view, mixed graphs generalize both graphs and digraphs. Given three natural numbers  $r$ ,  $z$  and  $k$ , the Degree/Diameter problem for mixed graphs asks for the largest possible number of vertices  $n(r, z, k)$  in a mixed graph with maximum undirected degree  $r$ , maximum directed out-degree  $z$  and diameter  $k$ . The study of this problem has deserved much attention in the last decades either for graphs and digraphs, but not for mixed graphs. In this talk we present some recent advances regarding this problem and we show some necessary conditions for the existence of these mixed graphs when their order is close to the Moore bound. Most of these conditions are related with the factorization in  $\mathbb{Q}[x]$  of the characteristic polynomial of these extremal mixed graphs.

24. Martin Mačaj, Comenius University, Bratislava, Slovakia. USA

**Title:** On the Defect of Vertex-transitive Graphs of Given Degree and Diameter

**Abstract:** We consider the problem of finding largest vertex-transitive graphs of given degree and diameter. Using two classical number theory results due to Niven and Erdős, we prove that for any fixed degree  $\Delta \geq 3$  and any positive integer  $\delta$ , the order of a largest vertex-transitive  $\Delta$ -regular graph of diameter  $D$  differs from the Moore bound by more than  $\delta$  for (asymptotically) almost all diameters  $D \geq 2$ . We also obtain an estimate for the growth of this difference, or defect, as a function of  $D$ . This talk is based on a joint work with G. Exoo, R. Jajcay and J. Širáň.

25. Diego Antonio González Moreno, Departamento de Matemáticas Aplicadas y Sistemas, Universidad Autónoma Metropolitana - Cuajimalpa

**Title:** On the Rainbow  $k$ -connectivity of Moore Cages of Girth Six

**Abstract:** Let  $G$  be an edge-colored graph. A path  $P$  of  $G$  is said to be *rainbow* if no two edges of  $P$  have the same color. Let  $G$  be a  $k$ -connected graph. An edge-coloring of  $G$  is a *rainbow  $k$ -coloring* if for any two distinct vertices  $u$  and  $v$  of  $G$  there are at least  $k$  internally vertex-disjoint rainbow  $(u; v)$ -paths. The *rainbow  $k$ -connectivity*,  $rc_k(G)$ , of a  $k$ -connected graph  $G$  is the minimum integer  $j$  such that there exists a rainbow  $k$ -coloring of  $G$  with  $j$  colors. A  $(k; g)$ -*cage* is a  $k$ -regular graph with girth  $g$  and the minimum possible order. A cage is said to be a *Moore cage* if the order of  $G$  attains the Moore bound. In this talk we show that if  $G$  is a Moore cage of girth 6, then  $k \leq rc_k(G) \leq k^2 - k + 1$ . It is also proved that the rainbow 3-connectivity of the Heawood graph is 6 or 7. The talk is a joint work with Camino Balbuena, Julián Fresán and Mika Olsen.

26. Gabriela Araujo-Pardo, Instituto de Matemáticas, Universidad Nacional Autónoma de México, Campus Juriquilla, Querétaro. México.

**Title:** On Mixed Cages

**Abstract:** In this talk we generalize the concept of *cage* and *directed cage*, introducing the idea of *mixed cage*. Recall that a cage is a regular graph of fixed girth (length of the minimal cycle) and minimum order. On the other hand, a *directed cage* is a directed regular graph (in each vertex emerge and inside the same number of arcs) of fixed "directed" girth (length of the minimal "directed" cycle) and minimum order.

We introduce the concept of *Mixed Cage*:  $A[z; r; g]$ -mixed cage is a mixed graph  $G$ ,  $z$ -regular by arcs,  $r$ -regular by edges, girth  $g$  (length of the minimal "mixed" cycle) and minimum order. First, we prove the existence of  $[z; r; g]$ -mixed cages and give families of them for some specific values, we give also lower and upper bounds for some parameters.

One motivation to us to study "Mixed Cages" is related with the study of "Mixed Moore Graphs", the research in this topic starts approximately 30 years ago, when in 1979 Bozak generalized the concept of Moore graphs and directed Moore graphs to Mixed Moore graphs (see [1] for more information of this topic); and with this idea we propose the generalization of cages and directed cages to mixed cages. This work is joined with: César Hernández Cruz and Juan José Montellano-Ballesteros.

[1] M. Miller, J. Širáň, *Moore graphs and beyond: A survey of the degree/diameter problem* Electronic Journal of Combinatorics, 20-2 (2013), DS14v2).

27. Charl Ras, School of Mathematics and Statistics, University of Melbourne

**Title:** The Computational Complexity of Geometric 2-connected Steiner Network Problems

**Abstract:** The geometric 2-connected Steiner network problem asks for a shortest bi-connected network spanning a given set of terminals in a 2-dimensional normed vector space, where additional nodes (called Steiner points) may be introduced anywhere

in the plane. Even without Steiner points, this problem is known to be NP-hard in the Euclidean plane. By reducing the problem to the Hamiltonian cycle problem in 2-connected planar bipartite graphs, we show that the geometric 2-connected Steiner network problem is NP-hard for any specific  $p$ -norm, where  $1 \leq p \leq 2$  or  $p = \infty$ . These values of  $p$  include the Euclidean and rectilinear (Manhattan) norms. Our construction shows that the problem is NP-hard even when Steiner points are not allowed.

28. Jana Šiagiová, Slovak University of Technology, Bratislava, Slovakia

**Title:** Approaching the Moore Bound for Diameter 3 Asymptotically by Cayley Graphs

**Abstract:** An independent set of a bipartite graph is called *balanced* if it contains exactly half its elements in each partite set. The problem of finding large balanced independent sets has applications in interconnection networks and coding theory, and we will use them here to construct upper and lower bounds for the vertex-isoperimetric number of a graph, which is a measure of how well-connected the graph is. In particular, we will closely investigate these results for the point-hyperplane incidence graph of  $PG(n, q)$ , and discuss exact solutions for small  $q$  as well as analytic solutions for large  $q$ .

29. Muhammad Adib Surani, School of Mathematics and Statistics, University of Melbourne, Australia

**Title:** The Isoperimetric Problem in Block Designs

**Abstract:** An independent set of a bipartite graph is called *balanced* if it contains exactly half its elements in each partite set. The problem of finding large balanced independent sets has applications in interconnection networks and coding theory, and we will use them here to construct upper and lower bounds for the vertex-isoperimetric number of a graph, which is a measure of how well-connected the graph is. In particular, we will closely investigate these results for the point-hyperplane incidence graph of  $PG(n, q)$ , and discuss exact solutions for small  $q$  as well as analytic solutions for large  $q$ .

30. Li-Da Tong, Department of Applied Mathematics, National Sun Yat-sen University, Taiwan

**Title:** Rearrangeable Nonblocking Multicast Banyan-type Networks with Crosstalk Constraints

**Abstract:** In the talk, the rearrangeability of  $f$ -cast Banyan-type networks with crosstalk constraints and  $f = 2^j$  has been well studied comprehensively. The necessary and sufficient conditions of the numbers of planes needed for being rearrangeable nonblocking, proposed in the talk, are extended for multicast connection requests, while the previous study results (in Chen, Tong, and Huang, Rearrangeable Nonblocking Optical Interconnection Network Fabrics With Crosstalk Constraints, IEEE/ACM

Trans. Networking, 2010, pp.1413-1421) are just for a special case for  $f = 1$  and one-to-one connection requests. The talk is mainly based on the joint works with Yun-Ruei Li.

31. James Tuite, Department of Mathematics and Statistics, Open University, UK

**Title:** Large Cayley Graphs of Fixed Diameter and Arbitrary Degree

**Abstract:** Much of the research on the degree/diameter problem has been devoted to establishing estimates for the maximum possible order of Cayley graphs of given diameter and arbitrary degree. Some of the best constructions, including a general construction for all diameters  $k \geq 3$ , use for the underlying group the semidirect product of a  $k$ -fold direct product of an abelian group with a cyclic group, where the cyclic group acts by permuting coordinates. In this talk, I will present a generalisation of these constructions that are the result of joint work with Grahame Erskine, which provide dramatically improved lower bounds.

32. Guillermo Pineda-Villavicencio, Centre for Informatics and Applied Optimisation, Federation University Australia, Australia

**Title:** On the Maximum Order of Graphs Embedded in Surfaces and Related Problems

**Abstract:** The maximum number of vertices in a graph of maximum degree  $\Delta \geq 3$  and fixed diameter  $k \geq 2$  is upper bounded by  $(1 + o(1))(\Delta - 1)^k$ . If we restrict our graphs to certain classes, better upper bounds are known. For instance, for the class of trees there is an upper bound of  $(2 + o(1))(\Delta - 1)^{\lfloor k/2 \rfloor}$  for a fixed  $k$ . The main result of this talk is that graphs embedded in surfaces of bounded Euler genus  $g$  behave like trees, in the sense that, for large  $\Delta$ , such graphs have orders bounded from above by

$$\begin{cases} c(g + 1)(\Delta - 1)^{\lfloor k/2 \rfloor} & \text{if } k \text{ is even} \\ c(g^{3/2} + 1)(\Delta - 1)^{\lfloor k/2 \rfloor} & \text{if } k \text{ is odd} \end{cases}$$

where  $c$  is an absolute constant. This result represents a qualitative improvement over all previous results, even for planar graphs of odd diameter  $k$ . With respect to lower bounds, we construct graphs of Euler genus  $g$ , odd diameter  $k$ , and order  $c(\sqrt{g} + 1)(\Delta - 1)^{\lfloor k/2 \rfloor}$  for some absolute constant  $c > 0$ . Our results answer in the negative a question of Miller and Širáň (2005).

This first part of the talk is based on the paper (E. Nevo, G. Pineda-Villavicencio and D. R. Wood, On the maximum order of graphs embedded in surfaces, *Journal of Combinatorial Theory, Series B* 119 (2016), 28-41).

In the second part of the talk, I want to draw the attention to a related problem: given a graph embedded in a surface of fixed genus and diameter  $k$ , can we find a constant-sized subset of vertices within which every other vertex is at distance half the diameter? This question was answered affirmatively by the paper (G. Borradaile and E. W. Chambers, Covering nearly surface-embedded graphs with a fixed number of balls, *Discrete Computational Geometry*, 51 (2014), no. 4, 979-996).

33. Jinjiang Yuan, School of Mathematics and Statistics, Zhengzhou University, China

**Title:** Unary NP-hardness of Minimizing the Number of Tardy Jobs with Deadlines

**Abstract:** In the single machine scheduling problem where the goal is to minimize the number of tardy jobs with deadlines, we are given a single machine and  $n$  jobs  $J_1, J_2, \dots, J_n$  to be scheduled. Each job  $J_j$  has a processing time  $p_j \geq 0$ , a due date  $d_j$ , and a deadline  $\bar{d}_j$ . In a feasible schedule, a job may be completed after its due date, but not after its deadline. A schedule  $\sigma$  is indicated by the permutation  $J_{\sigma(1)}, J_{\sigma(2)}, \dots, J_{\sigma(n)}$  of the  $n$  jobs, where  $J_{\sigma(j)}$  is the job processed at the  $j$ -th position in  $\sigma$ . Given a schedule  $\sigma = (J_{\sigma(1)}, J_{\sigma(2)}, \dots, J_{\sigma(n)})$ , we use  $C_{\sigma(j)}(\sigma) = p_{\sigma(1)} + p_{\sigma(2)} + \dots + p_{\sigma(j)}$  to denote the completion time of job  $J_{\sigma(j)}$ . Job  $J_j$  is tardy in  $\sigma$  if  $C_j(\sigma) > d_j$  and on time if  $C_j(\sigma) \leq d_j$ . Moreover, we use  $U_j(\sigma)$  to denote the tardy index of job  $J_j$  in schedule  $\sigma$ , that is,  $U_j(\sigma) = 1$  if  $J_j$  is tardy in  $\sigma$  and  $U_j(\sigma) = 0$  if  $J_j$  is on time in  $\sigma$ . Then the number of tardy jobs of schedule  $\sigma$  is given by  $\sum_{j=1}^n U_j(\sigma)$ . A schedule  $\sigma$  is called feasible (subject to the deadlines) if  $C_j(\sigma) \leq \bar{d}_j$  for each  $j = 1, 2, \dots, n$ . The goal of the problem is to find a feasible schedule  $\sigma$  such  $\sum_{j=1}^n U_j(\sigma)$  is minimized. Using the standard notation, the problem is denoted by  $1|\bar{d}_j|\sum U_j$ .

Lawler showed in 1983 that problem  $1|\bar{d}_j|\sum U_j$  is binary NP-hard. But the exact complexity (unary NP-hard or pseudo-polynomial-time solvability) of the problem is a long standing open problem. We show in this paper that the problem is unary NP-hard. Our research also implies that the scheduling problem for finding an optimal schedule to minimize the number of tardy jobs which also satisfies the restriction of deadlines is unary NP-hard. As consequences, some multi-agent scheduling problems related to minimizing the number of tardy jobs and the maximum lateness are unary NP-hard.

34. Cunquan Zhang, West Virginia University, USA

**Title:** Integer 11-flows of Signed Graphs

**Abstract:** In 1983, Bouchet proposed a conjecture that every flow-admissible signed graph has a nowhere-zero 6-flow. Recently, DeVos proved that every such signed graph admits a nowhere-zero 12-flow. In this talk, we will present a recent improvement of DeVos's result for bridgeless signed graphs, and show that every bridgeless flow-admissible signed graph admits a nowhere-zero 11-flow.

35. Shenggui Zhang, Northwestern Polytechnical University, China

**Title:** Rainbow Cliques and Matchings in Edge-colored Graphs

**Abstract:** An edge-colored graph is one in which each edge is assigned a color. A subgraph of an edge-colored graph is called rainbow if all of its edge have distinct colors. In this talk, I will mainly introduce some results on the existence of rainbow cliques and matchings in edge-colored graphs.

36. Xiao-Dong Zhang, Shanghai Jiao Tong University, China

**Title:** Laplacian Eigenvalues and Consensus Problems on Small-world Networks

**Abstract:** In this talk, we introduce relationship between the consensus problems and the algebraic connectivity of the small-world networks. We give a mathematical rigorous estimation of the lower bound for the algebraic connectivity of the small-world network, which explains why the consensus problems on the small-world network have a ultrafast convergence rate and how much it can be improved. Moreover, we also prove that the small-world networks possess large clustering coefficient and small diameter. This works are joined with Lei Gu, Huiling Huang, Qing Zhou.

37. Jin-Xin Zhou, Department of Mathematics, Beijing Jiaotong University

**Title:** Tetravalent Half-arc-transitive  $p$ -Graphs

**Abstract:** A graph is *half-arc-transitive* if its automorphism group acts transitively on vertices and edges, but not on arcs. Let  $p$  be a prime. A graph is called a  *$p$ -graph* if it is a Cayley graph of order a power of  $p$ . In this talk, I will introduce our recent work on characterization of tetravalent edge-transitive  $p$ -graphs, construction of tetravalent half-arc-transitive non-normal  $p$ -graphs, and characterization of a special class of tetravalent half-arc-transitive non-metacirculant  $p$ -graphs, where  $p$  is an odd prime.

## List of participants

Patrick Andersen	The University of Melbourne, Australia
Gabriela Araujo-Pardo	Universidad Nacional Autonoma de Mexico, Mexico
Martin Bachraty	Slovak University of Technology, Slovakia
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Sun-Yuan Hsieh	National Cheng Kung University, Taiwan, R.O.C.
Leif Jorgensen	Aalborg University, Denmark
Liyang Kang	Shanghai University, China
Sandi Klavžar	University of Ljubljana, Slovenia & University of Maribor, Slovenia
Martin Knor	Slovak University of Technology, Slovakia
Jack Koolen	University of Science and Technology of China
Felix Lazebnik	University of Delaware, USA
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