Existence of Positive Solutions with a Prescribed Singular Set for Fractional Yamabe Problem

Weiwei Ao

Wuhan University

Abstract: We consider the problem of the existence of positive solutions with prescribed isolated singularities of the fractional Yamabe problem. Near each singular point, these solutions are approximated by the Delaunay-type singular solution which has been studied recently by De la Torre, Del Pino, Mar Gonzalez and J.C. Wei. Away from the singular points, these solutions are approximated by the summation of the Green's function. This result is the analogous result for the classical Yamabe problem studied by Mazzeo and Pacard (1999). This is a joint work with De la Torre, Mar Gonzalez and J.C. Wei.

Existence and Uniqueness of Vortex Patch for Two Dimensional Euler Equation

Daomin Cao

Institute of Applied Mathematics, Chinese Academy of Science

Abstract: In this talk, the speaker will talk about the results obtained in recent years on the existence and uniqueness of vortex patch for two dimensional imcompressible Euler equation. It turns out that the number of vortex patch concentrated near some points is closely related to the number of critical points of the so-called Kirchhoff-Routh function, which is determined by the Green's function of the domain on which the equation is set on. The talk is based on results of papers with Yuxia Guo, Shuangjie Peng and Shusen Yan.

Estimates of Eigenvalues for a Class of Bi-subelliptic Operators

Hua Chen

Wuhan University

Abstract Let $X = (X_1, X_2, \cdots, X_m)$ be a system of real smooth vector fields defined on a bounded open domain $\Omega \subset \mathbb{R}^n$ with smooth boundary $\partial\Omega$ which is non-characteristic for X. If X satisfies the Hormander's condition, then the vector fields is finitely degenerate and the sum of square operator $\triangle_X = \sum_{j=1}^m X_j^2$ is a subelliptic operator. In this talk, let λ_k be the k-th Dirichlet eigenvalue for the bi-subelliptic operator $\triangle_X^2$ on Ω, we shall use the subelliptic estimates to give the explicit lower bounds of λ_k, which is the extension of the classical result for bi-harmonic operator.

A Theory Connecting Mean Field Equations on Tori and Painlevé VI Equation

Zhijie Chen
Tsinghua University

Abstract: We develop a new theory to connect two areas: (i) Painlevé VI equation and (ii) the mean field equation with multiple singular sources on tori. This gives a geometric interpretation of solutions to Painlevé VI equation with some special parameters. We establish this theory by applying a generalized Lamé equation (a linear ODE). As an application, we obtain a sharp existence result for the mean field equation on tori. This is a joint work with Professors Chang-Shou Lin and Ting-Jung Kuo.

Hölder Estimates for Solutions of a MEMS Equation

Juan Diego Davila
University of Chile

Abstract: We prove sharp Hölder estimates for sequences of positive solutions of a nonlinear elliptic problem with negative exponent. As a consequence, we prove the existence of solutions with isolated ruptures in a bounded convex domain in two dimensions.

This is joint work with Kelei Wang (Wuhan University) and Juncheng Wei (University of British Columbia).

Singularity Formation for the Two-dimensional Harmonic Map Flow into S^2

Manuel del Pino
University of Chile

Abstract: We construct finite time blow-up solutions to the 2-dimensional harmonic map flow into the sphere S^2,

$$\begin{aligned} u_t = \Delta u + |\nabla u|^2 u \quad & \text{in } \Omega \times (0,T) \\ u = \varphi \quad & \text{on } \partial\Omega \times (0,T) \\ u(\cdot,0) = u_0 \quad & \text{in } \Omega \end{aligned}$$

where Ω is a bounded, smooth domain in $\mathbb{R}^2$ and $u : \Omega \times (0,T) \to S^2$, $u_0 : \bar{\Omega} \to S^2$, smooth, $\varphi = u_0\big|_{\partial\Omega}$. Given any points $q_1, \ldots, q_k$ in the domain, we find initial and boundary data so that the solution blows-up precisely at those points. The profile around each point is close to an asymptotically singular scaling of a 1-corrotational harmonic map. We prove stability of this phenomenon if $k = 1$. This is joint work with Juan Dávila and Juncheng Wei.

Variational Methods for Strongly Indefinite Problems

Yanheng Ding

Institute of Mathematics, Chinese Academy of Sciences

Abstract: Consider the following general nonlinear system

$$Au = N(u) \quad (1)$$

where H is a Hilbert space, A is a self-adjoint operator, and N is a (nonlinear) gradient operator. Typical example are Dirac equations and reaction-diffusion systems where $\sigma(A)$ (the spectrum) is unbounded from below and above, and particularly, $\sigma_e(A) \cap \mathbb{R}^{\pm} \neq \emptyset$. The talk focus on

- to establish general variational setting for (1) by using the operator interpolation theory;
- certain critical point theory;
- the existence, concentration and exponential decay for semi- classical solutions of of Dirac equation and the reaction-diffusion systems, etc.;
- bifurcation of Dirac equation on spin manifolds.

Optimality in Critical and Subcritical Inequalities: Flows, Linearization and Entropy Methods

Jean Dolbeault

Centre national de la recherche scientifique & Université Paris-Dauphine

Abstract: Symmetry breaking results for optimal functions in critical and subcritical functional inequalities involving weights can be obtained by a local analysis (linear instability). Symmetry results for optimal functions and rigidity results for solutions of nonlinear elliptic PDEs are by nature global. Nonlinear flows are central for their interpretation. The lecture will be centered on linearization, symmetry breaking and large time asymptotics of the flows on the one hand, and on the role of the flows and their linearization for monotonicity and optimality issues in interpolation inequalities, with and without weights, on the other hand.

Most of the recent results presented in this talk have been obtained in collaboration with M.J. Esteban, M. Loss and M. Muratori.

Equilibria of Point-Vortices on Closed Surfaces

Pierpaolo Esposito

University of Rome 3

I will discuss the existence of equilibrium configurations for the Hamiltonian point-vortex model on a closed surface. Its topological properties determine the occurrence of three distinct situations, corresponding topologically to the sphere, to the real projective plane and to the remaining cases. As a by-product, new existence results are obtained for the singular mean-field equation with exponential nonlinearity.

Joint work with T. D'Aprile.

Bifurcation, Rigidity and Symmetry for Critical and Subcritical Inequalities

Maria J. Esteban

Centre national de la recherche scientifique & University Paris-Dauphine

Abstract: In this talk will be presented several results about symmetry and symmetry breaking for optimizers of critical and subcritical functional inequalities. They will be shown to be linked to rigidity results for solutions of nonlinear elliptic PDEs with and without weights. Some interesting bifurcation phenomena will help to understand the different situations that can arise.

The works presented in this talk have been obtained in collaboration with J. Dolbeault, M. Loss and M. Muratori.

Unbounded Periodic Solutions to a Class of Overdetermined Boundary Value Problems

Mouhamed Moustapha Fall

The African Institute for Mathematical Sciences, Senegal

Abstract: In 1971, Serrin proved that certain overdetermined boundary value elliptic equations, considered on underlying regular bounded domains, must have radially symmetric solutions. This result parallels Alexandrov rigidity theorem, stating that "the only embedded constant mean curvature surface in euclidean is the round sphere". In this talk, we discuss the existence of unbounded periodic solutions to a classical overdetermined problem, with underlying periodic domains are of the type: Bands in 2D, Slabs and Unduloids in 3D. Some applications will be also given.

Nonradial Solutions for Equations with Critical Exponents in $\mathbb{R}^N$

Yuxia Guo

Tsinghua University

Abstract: In this talk, we are concerned with the existence, multiplicity and local uniqueness of nonradial solutions for equations with critical exponent, including polyharmonic equations and fractional operator equations. Our results indicate that the presence of the curvature function will affect the existence of the solutions.

Existence Theorems on Vortex Solutions in The ABJM Model

Xiaosen Han

Henan University

Abstract: In this talk several existence theorems concerning the vortex solutions for some nonlinear PDE systems arising in the ABJM model will be presented. Firstly, we recall some background for the ABJM model. Secondly, we state a series of existence theorems for the related PDE systems. Thirdly, the proofs of the main results are sketched. At last some related open problems will be mentioned.

Variational Approach to Mixed Type Solutions in SU(3)-CS Model

Namkwon Kim

Chosun University

Abstract: The nontopological solutions and mixed type solutions in SU(3) Chern-Simons model over $\mathbb{R}^2$ are subtle one. There are variational functionals for them but they are not coercive and hence we have to deal with mountain pass type critical points. By this reason, it is believed that variational approach to nontopological and mixed type solutions in $\mathbb{R}^2$ is more delicate than other approaches. We deal with some details related with the variational formulation for the mixed type solutions and some related problems.

Existence and Non-existence for the Mean Field Equations at Critical Parameter 16π

Ting-Jung Kuo

Taiwan University

Abstract: A conjecture about the mean field equation

$$\Delta u + e^u = 8n\pi\delta_0$$

on a flat torus E_τ is the non-existence of solutions if $\tau \in i\mathbb{R}^+$. For any $n \in \mathbb{N}_{\geq 2}$, this conjecture seems very challenging from the viewpoint of PDE theory. In order to solve this conjecture, a premodular form $Z^{(n)}_{r,s}(\tau)$ was introduced by Wang and Lin, and is used to give necessary and sufficient conditions for the existence of solutions. In this talk, we will talk about the conjecture for $n = 2$ (i.e. at critical parameter 16π). In turn, we could apply this non-existence result to obtain the structure of zeros of the premodular form $Z^{(2)}_{r,s}(\tau)$. As a consequence, we obtain the existence of solutions for $n = 2$ if $\tau = \frac{1}{2} + ib$ and $b > b^*$ for some $b^* \in (\frac{\sqrt{3}}{2}, \frac{6}{5})$. This is a joint work with Z. Chen and C.S. Lin.

Degree Counting for Toda System of Rank Two: One Bubbling

Youngae Lee

Taiwan University

Abstract: In this talk, we study the degree counting formula of the rank two Toda system with simple singular sources. The key step is to derive the degree formula of the shadow system, which arises from the bubbling solutions as one of parameters crosses 4π. In order to compute the topological degree of the shadow system, we need to find some suitable deformation. During this deformation, we shall deal with new difficulty arising from the new phenomena: blow up does not necessarily imply concentration of mass. This phenomena occurs due to the collapsing of singularities. This talk is based on the joint works with Prof. Chang-Shou Lin, Prof. Gabriella Tarantello, Prof. Juncheng Wei, Prof. Lei Zhang, and Dr. Wen Yang.

Virial Theorem and Eigenvalue Estimate of Nonlinear Schrödinger Equations

Tai-Chia Lin

Taiwan University

Abstract: Conventionally, the virial theorem (about the ratio of the total kinetic energy and the total potential energy) is useful to get the eigenvalue estimate of linear Schrodinger equations, which is important in quantum mechanics. Until now, there is no general method to study the eigenvalue estimate of nonlinear Schrodinger equations. Here we use the Pohozaev identity to develop the virial theorem and eigenvalue estimate of nonlinear Schrodinger equations with square-root and saturable nonlinearity, which describe photorefractive mediums, narrow-gap semiconductors

and graphene metamaterials. Theoretical results can be supported by numerical experiments. For solitons of photorefractive media, our results show the relation between the beam-coupling constant and the eigenvalue which represents the propagation constant.

Improvements and Generalizations of Clark's Theorem and Applications

Zhaoli Liu

Capital Normal University

Abstract: In critical point theory, Clark's theorem states as follows. Let X be a Banach space, $\Phi \in C^1(X, \mathbb{R})$. Assume Φ satisfies the (PS) condition, is even and bounded from below, and $\Phi(0) = 0$. If for any $k \in \mathbb{N}$, there exists a k-dimensional subspace X^k of X and $\rho_k > 0$ such that $\sup_{X^k \cap S_{\rho_k}} \Phi < 0$, where $S_\rho = \{u \in X | \, \|u\| = \rho\}$, then Φ has a sequence of critical values $c_k < 0$ satisfying $c_k \to 0$ as $k \to \infty$. We improve Clark's theorem by showing that under the assumptions of Clark's theorem Φ has a sequence of critical points u_k such that $\Phi(u_k) \leq 0$ and $u_k \to 0$ as $k \to \infty$. We also generalize Clark's theorem by replacing the C^1 smoothness, the boundedness from below, and the (PS) condition with weaker assumptions respectively. The new results produce infinitely many solutions to various nonlinear equations under quite general conditions. (This is joint work with Shaowei Chen and Zhi-Qiang Wang.)

Minimal Energy Solutions and Bifurcation Results for a Weakly Coupled Nonlinear Schrödinger System

Rainer Mandel

Karlsruhe Institute of Technology

In this talk I intend to give a panoramic view on existence results for fully nontrivial solutions of the nonlinear Schrödinger system

$$\begin{aligned} -\Delta u + \quad u &= u^3 + buv^2 \qquad \text{in } \mathbb{R}^n, \\ -\Delta v + \omega^2 v &= v^3 + bvu^2 \qquad \text{in } \mathbb{R}^n, \\ u, v \in H^1(\mathbb{R}^n), \quad n \in \{1,2,3\}, \quad \omega &> 0, b \in \mathbb{R} \end{aligned}$$

which have been obtained during the past ten years, see [1, 2, 3, 4, 5, 6, 7]. The focus will be set on a comparison of the methods coming from bifurcation theory and constrained minimization techniques. Amongst other things I will show that in case $n \in \{2,3\}$ positive solutions exist and converge

to a solution of some optimal partition problem as the coupling parameter b tends to $-\infty$ whereas this phenomenon does not occur when $n = 1$.

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Nonlinear Time-harmonic Maxwell Equations in a Bounded Domain

Jarosław Mederski

Institute of Mathematics, Polish Academy of Sciences

Abstract: The search for time-harmonic solutions of nonlinear Maxwell equations in the absence of charges and currents leads to the following equation

$$\begin{cases} \nabla \times \left(\mu(x)^{-1}\nabla \times E\right) - \omega^2 \varepsilon(x)E = f(x,E) & \text{in } \Omega \\ \nu \times E = 0 & \text{on } \partial\Omega, \end{cases}$$

for the field $E : \Omega \to \mathbb{R}^3$, where $\Omega \subset \mathbb{R}^3$ is a bounded Lipschitz domain with exterior normal $\nu : \partial\Omega \to \mathbb{R}^3$, $\varepsilon(x) \in \mathbb{R}^{3\times 3}$ is the (linear) permittivity tensor of the material, and $\mu(x) \in \mathbb{R}^{3\times 3}$ denotes the magnetic permeability tensor. The nonlinearity $f : \Omega \times \mathbb{R}^3 \to \mathbb{R}^3$ comes from the nonlinear polarization. If $f = \nabla_E F$ is a gradient then this equation has a variational structure. Our goal is to present ground state and bound state solutions for superlinear and subcritical nonlinearities f, e.g. of the form $\Gamma(x)|E|^{p-2}E$ with $2 < p < 2^* = 6$, obtained jointly with Thomas Bartsch. Moreover we discuss the critical case when $p = 6$.

Infinite-time Bubbling in the Critical Nonlinear Heat Equation

Monica Musso

Pontificia Universidad Católica de Chile

Abstract: In this talk I will present two results concerning construction of infinite time bubbling solutions for critical nonlinear heat equations of Fujita type.

The first result is on a smooth bounded domain Ω in $\mathbb{R}^n$, $n \geq 5$. We consider the classical semilinear heat equation at the critical Sobolev exponent

$$u_t = \Delta u + u^{\frac{n+2}{n-2}} \quad \text{in } \Omega \times (0,\infty), \quad u = 0 \quad \text{on } \partial\Omega \times (0,\infty).$$

Given any integer $k \geq 1$, we prove the existence of a positive smooth solution $u(x,t)$ which blows-up by bubbling in infinite time near k points $q_1, \ldots, q_k$ in Ω. More precisely, for large time t, u takes the approximate form

$$u(x,t) \approx \sum_{j=1}^{k} \alpha_n \left(\frac{\mu_j(t)}{\mu_j(t)^2 + |x - \xi_j(t)|^2} \right)^{\frac{n-2}{2}}.$$

Here $\xi_j(t) \to q_j$ and $0 < \mu_j(t) \to 0$, as $t \to \infty$. We find that $\mu_j(t) \sim t^{-\frac{1}{n-4}}$ as $t \to +\infty$. This work is in collaboration with Manuel del Pino and Carmen Cortázar.

The second result is on the whole space $\mathbb{R}^3$. We construct a globally defined radially symmetric positive solution to

$$u_t = \Delta u + u^5, \quad \text{in} \quad \mathbb{R}^3 \times (0,\infty),$$

with $\lim_{r\to\infty} r^\gamma u(r,0) = A > 0$, for some $\gamma > 1$. We show that, as $t \to \infty$,

$$\|u(r,t)\|_\infty \sim \frac{1}{t^{\frac{\gamma-1}{2}}}, \quad \text{if} \quad 1 < \gamma < 2, \quad \|u(r,t)\|_\infty \sim \frac{1}{\sqrt{t}}, \quad \text{if} \quad \gamma > 2,$$

and

$$\|u(r,t)\|_\infty \sim \frac{\ln t}{\sqrt{t}}, \quad \text{if} \quad \gamma = 2.$$

This work is in collaboration with Manuel del Pino and Juncheng Wei.

On Laudau-Kolmogorov Inequalities for Dissipative Operators

Tohru Ozawa

Waseda University

Abstract. We revisit Kato's theory on Landau-Kolmogorov (or Kallman-Rota) inequalities for dissipative operators in an algebraic framework in a scalar product space. This is a joint-work with Masayuki Hayashi.

The Singular Liouville Problem in the Plane as Limit of Lane-Emden Problems: Asymptotics and Morse Index

Frank Pacella

University of Roma "Sapienza"

Abstract: We present recent results that show that the singular Liouville equation in the plane can be viewed as a limit of semilinear elliptic equations of Lane-Emden type as the exponent of the nonlinearity becomes very large. This is achieved through the study of the asymptotic behavior of families of sign changing solutions whose nodal line does not touch the boundary. In particular this phenomenum arises while studying radial sign changing solutions of Lane Emden problems. An accurate study of the spectrum of the linearized operator shows a relation between the Morse index of these solutions and that of a specific solution of the limit problem. As a consequence we get the existence of new nonradial sign changing solutions in the ball. The results are contained in some papers in collaboration with F. De Marchis-I.Ianni and M.Grossi-C. Grumiau.

Local Uniqueness and Periodicity Induced by Concentration

Shuangjie Peng

Central China Normal University

Abstract: We will talk about the following poly-harmonic equations with critical exponents:

$$(-\Delta)^m u = K(y)u^{\frac{N+2m}{N-2m}}, \quad u > 0 \quad \text{in } \mathbb{R}^N,$$

where $N > 2m+2, m \in \mathbb{N}_+, K(y)$ is positive and periodic in its first k variables $(y_1, \cdots, y_k)$, $1 \leq k < \frac{N-2m}{2}$. Under some conditions on $K(y)$ near its critical point, we prove the existence and local uniqueness of solutions with infinitely many bubbles. The local uniqueness result implies that some bubbling solutions preserve the symmetry of the scalar curvature $K(y)$. Moreover, we also show that the conditions imposed are optimal to obtain such results.

On the Existence of Solutions to Weakly Coupled Elliptic System with Critical Growth

Angela Pistoia

Universit di Roma "La Sapienza"

Abstract: We consider a critical weakly coupled elliptic systems in a domain D in $\mathbb{R}^N$ with $N = 3, 4$. We prove the existence of positive solutions which blow-up at one or more points in D provided some conditions are satisfied. The results have been obtained in collaboration with Nicola Soave and Hugo Tavares.

A Rigorous Derivation of the Nonlinear Euler-Heisenberg Model for Static Magnetic Fields

Eric Sere

University of Dauphine Paris

Abstract: This is joint work with Philippe Gravejat and Mathieu Lewin (2016, arXiv:1602.04047). The Euler-Heisenberg model provides a nonlinear system of equations for the electromagnetic field. The nonlinearity takes into account the interaction between the classical electromagnetic field and the quantum vacuum. It depends on a small coupling parameter and one recovers the linear Maxwell equations when this parameter is set to zero. In most situations the linear (Maxwell) approximation is extremely accurate, but nonlinear effects cannot be neglected in very strong fields, as for instance on the surface of some neutron stars. We give the first rigorous derivation of the Euler-Heisenberg magnetic energy in the semi-classical limit of slowly varying, time-independent, magnetic fields. The question of (slowly) time-varying fields remains open.

Normalized Solutions for Nonlinear Schrödinger Systems

Nicola Soave

Justus-Liebig-University of Giessen

Abstract: We consider the system of coupled elliptic equations

$$\begin{cases} -\Delta u - \lambda_1 u = \mu_1 u^3 + \beta u v^2 \\ -\Delta v - \lambda_2 v = \mu_2 v^3 + \beta u^2 v \end{cases} \quad \text{in } \mathbb{R}^3,$$

studying the existence of positive solutions satisfying the additional condition

$$\int_{\mathbb{R}^3} u^2 = a_1^2 \quad \text{and} \quad \int_{\mathbb{R}^3} v^2 = a_2^2.$$

Assuming that a_1, a_2, μ_1, μ_2 are positive fixed quantities, we prove existence results for different ranges of the coupling parameter β, which can take both positive and negative values. Our proofs are based upon minimax methods

($\beta > 0$) and upon the introduction of a suitable natural constraint ($\beta < 0$). This last technique can also be applied to treat the scalar NLS equation

$$\begin{cases} -\Delta u - \nu u = f(u) \quad \text{in } \mathbb{R}^N \\ u \in H^1(\mathbb{R}^N) \\ \int_{\mathbb{R}^N} u^2 = a^2, \end{cases}$$

and permits to give alternative and simpler proofs to known results. This talk is based on joint works with T. Bartsch (2016), and with T. Bartsch and L. Jeanjean (2015).

Geometric Aspects of Phase Separation

Susanna Terracini
Università di Torino

Abstract: Several physical phenomena can be described by a certain number of densities (of mass, population, probability, ...) distributed in a domain and subject to laws of diffusion, reaction, and *competitive interaction*. Whenever the competitive interaction is the prevailing phenomenon, the several densities can not coexist and tend to segregate, hence determining a partition of the domain *(Gause's experimental principle of competitive exclusion (1932))*. As a model problem, we consider the system of stationary equations

$$\begin{cases} -\Delta u_i = f_i(u_i) - \beta u_i \sum_{j \neq i} g_{ij}(u_j) \\ u_i > 0 \ . \end{cases}$$

The cases $g_{ij}(s) = \beta_{ij} s$ (Lotka-Volterra competitive interactions) and $g_{ij}(s) = \beta_{ij} s^2$ (gradient system for Gross-Pitaevskii energies) are of particular interest in the applications to population dynamics and theoretical physics respectively.

We will undertake the analysis of qualitative properties of solutions to systems of semilinear elliptic equations, whenever the parameter β, accounting for the competitive interactions, diverges to infinity. At the limit, when the minimal interspecific competition rate $\beta = \min_{ij} \beta_{ij}$ diverges to infinity, we find a vector $U = (u_1, \cdots, u_h)$ of functions with mutually disjoint supports: *the segregated states*: $u_i \cdot u_j \equiv 0$, for $i \neq j$, satisfying

$$-\Delta u_i = f_i(x, u_i) \qquad \text{whenever } u_i \neq 0 \ , \quad i = 1, \dots, h,$$

We will consider the follfowing apsects:

(1) Entire solutions of the competitive elliptic system:

$$\begin{cases} -\Delta u_i = -\sum_{j \neq i} u_i u_j^2 & \text{in } \mathbb{R}^N \\ u_i > 0 & \text{in } \mathbb{R}^N \end{cases} \qquad i = 1, \dots, k. \tag{0.1}$$

(2) Spiralling solutions in the non symmetrical case.

Global Attractor for the 3rd Order Lugiato-Lefever Equation on 1D Torus

Yoshio Tsutsumi
Kyoto University

Abstract: We consider the third order Lugiato-Lefever equation:

$$\partial_t u - \partial_x^3 u + i\alpha\partial_x^2 u + u + i|u|^2 u = f, \qquad t > 0, \quad x \in \mathbf{T}, \tag{0.2}$$

$$u(0,x) = u_0(x), \qquad x \in \mathbf{T}. \tag{0.3}$$

We assume that $2\alpha/3 \notin \mathbf{Z}$. Equation (0.2) appears as a mathematical model, for example, for Kerr frequency comb generation in a whispering gallery mode resonator, octave-spanning Kerr frequency comb in a micro-ring resonator, and cavity solitons in micro-ring resonator near zero group-dispersion. An increasing attention among theoretical and experimental physicists in that field has been paid to the role of third order dispersion, i.e. the third order derivative in (0.2). We show the existence of the global attractor in $L^2(\mathbf{T})$ for (0.2). Without damping and forcing terms, it has three conserved quantities, that is, the $L^2(\mathbf{T})$ norm, the momentum and the energy, but the leading term of the energy functional is not positive definite. So only the L^2 norm conservation is useful for (0.2) unlike the KdV and the cubic NLS equations. Therefore, it seems important and natural to construct the global attractor in $L^2(\mathbf{T})$. For the proof of the global attractor, we use the smoothing effect of cubic nonlinearity for the reduced equation. This is a joint work with Tomoyuki Miyaji, Meiji University.

Asymptotics in Coupled Nonlinear Schrödinger Equations with Large Mixed Couplings

Zhi-Qiang Wang
Tianjin University and Utah State University

Abstract: We discuss work on existence and qualitative property of positive solutions for coupled nonlinear Schrödinger equations. Depending upon the system being attractive or repulsive, solutions may tend to be component-wisely synchronized or segregated. We report recent work on the effect of mixed couplings for which coexistence of synchronization and segregation may occur, in particular, we examine the asymptotic behavior of least energy solutions for large mixed couplings of multi-scales.

Higher Order Excess Decay and Uniqueness of Blowing Down Limit

Kelei Wang
Wuhan University

Abstract: In this talk I will discuss some problems related to solutions with polynomial growth of the following elliptic system

$$\Delta u = uv^2, \quad \Delta v = vu^2, \quad u, v > 0 \text{ in } \mathbb{R}^n.$$

Using a blowing down analysis, we konw that the solution (after a rescaling) converges (up to a subsequence) to (Φ^+, Φ^-), the positive and negative part of a homogeneous harmonic polynomial. I will discuss a higher order excess decay estimate and show how to use this estimate to prove the uniqueness of the blowing down limit.

Global Minimizers of Allen-Cahn and Free Boundary Problems

Juncheng Wei
University of British Columbia

Abstract: I will first present a result on the existence of non-trivial global minimizers of Allen-Cahn equation in dimension 8. Then I will discuss related issues on two free boundary problems:

$$\Delta u = 0 \text{ in } \Omega$$

$$|\nabla u| = 1 \text{ on } \partial\Omega$$

where Ω consists of either one component (the one-phase free boundary problem) or two components. (Joint work with Y. Liu, Kelei Wang.)

Title: TBA

Wenming Zou
Tsinghua University