Rotational beta expansion and self-similar tiling

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ABSTRACT: We generalize beta expansions to higher dimension involving an action of isometry and study their basic properties, such ergodicity and soficness. In particular, sofic cases give a construction of a large family of self-similar tilings. It is noteworthy that we have an easy way to construct self-similar polygonal tilings with $2n$-fold (diffractive) symmetry for any n. This is a joint work with Jonathan Caalim.

Porosities of fractal percolation

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ABSTRACT: We study porosities in the fractal percolation process. We show that, almost surely at almost all points with respect to the natural measure, the mean porosities of the set and the natural measure exist and are equal to each other for all parameter values outside of a countable exceptional set. As a corollary, we obtain that, almost surely at almost all points, the lower porosities of the set and the natural measure are equal to zero, whereas the upper porosities obtain their maximum values.

Some fractal tilings and measures

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ABSTRACT: First, I show a fractal tile of pinwheel type which seems the first truly non-crystallographic self-similar plane tile with fractal boundaries. This is joint work with D. Mekhontsev and A. Tetenov.

Then I present an approach to the parametric family of Bernoulli convolutions based on their two-dimensional density. Recent work on algebraic numbers by Breuillard and Varju, and by Hare and Sidorov has revealed new parameters for which the dimension of the Bernoulli measure equals one. I try to explain why a more concrete approach to the whole family has the potential to get even better results.

Both parts of the lecture are intended to be comprehensible, non-technical and supported by computer illustrations.

Diophantine approximation properties of powers of real numbers

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ABSTRACT: Denote by $\{\cdot\}$ the fractional part of a real number. The Diophantine approximation properties of the sequence $(\{x^n\})_{n\geq 1}$ are studied. Some Hausdorff dimension results on good approximation, both in asymptotic way and uniform way, and on bad approximation are obtained.

Capacity of range of SRW

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ABSTRACT: We study the capacity of the range $\{X_0, \ldots, X_n\}$ of a simple random walk X on Z^d. We prove the weak law of large numbers for $d = 4$ and disprove the weak law of large numbers for $d = 3$, conjectured by Asselah, Schapira and Sousi.

On the k-abelian complexity of the Cantor sequence

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ABSTRACT: We prove that for every integer $k \geq 1$, the k-abelian complexity function of the Cantor sequence $c = 101000101\cdots$ is 3-regular. Moreover, for every integer $k, \ell \geq 1$, the k-abelian complexity function of the Cantor-like sequence generated by $\sigma := (10^\ell 1, 0^{\ell+2})$ is also 3-regular.

Stepped surfaces and Rauzy fractals related to minimal polynomials of hyperbolic cubic unit integers

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ABSTRACT: S. Ito and M. Furukado classified the minimal polynomials of hyperbolic cubic unit integers into four cases, and found the automorphisms on the

free group of rank 3 related to their companion matrices. They tried to construct stepped surfaces and Rauzy fractals induced from the automophisms, but they pointed out that some case was difficult.

In my talk, we recall the result of S. Ito and M. Furukado, and construct stepped surfaces and Rauzy fractals in the "difficult" case.

On union of spectrum for Sturm Hamiltonian

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ABSTRACT: We study the union of spectrum for Sturm Hamiltonians with all frequency. Let $A = [0,1] \cap Q^c$, $V > 4$. We study the set $S_V = \bigcap_{\alpha \in A} \sigma(H_{V,\alpha})$, where $H_{V,\alpha}$ is the Schrodinger operator with coupling V and frequency α, $\sigma(H_{V,\alpha})$ is spectrum of $H_{V,\alpha}$. We show that there exists an at most countable set Λ such that $S_V = [-2,2] \cup [V-2, V+2] - \Lambda$ and S_V contain intervals.

Some questions related to weighted Birkhoff averages

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ABSTRACT: We consider weighted Birkhoff sums of the form $\sum_{n=0}^{N-1} w_n f(T^n x)$ and related objects. These sums can be studied, basing on the spectral properties of the sequence of weights (w_n). Interesting weights (w_n) include q-multiplicative sequences and and oscillating sequences. Multifractal analysis of some of these sums can also be made.

Lyapunov exponents of non-negative matrices

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ABSTRACT: Let $\mathbf{M} = (M_1, \cdots, M_k)$ be a tuple of $d \times d$ non-negative matrices. Suppose that $\sum_{i=1}^k M_i$ is an irreducible matrix. With the help of symbolic dynamics, we give an algorithm for checking whether or not $\mathbf{M}$ possesses the following property: there exist two constants $\lambda \in \mathbb{R}$ and $C > 0$ such that for any $n \in \mathbb{N}$, and any $i_1, \cdots, i_n \in \{1, \cdots, k\}$, either $M_{i_1} \cdots M_{i_n} = 0$ or $C^{-1} e^{\lambda n} \leq \|M_{i_1} \cdots M_{i_n}\| \leq C e^{\lambda n}$, where $\|\cdot\|$ is a matrix norm. As applications, we are able to check the absolute continuity of a class of overlapping self-similar measures on $\mathbb{R}$, and the dimensional regularity of a class of sofic affine-invariant sets in the plane.

The talk is based on joint work with Chiu-Hong Lo and Shuang Shen.

A Realization of Optimum Binary Spreading Sequences of Markov Chains Based on Discretized β-transformations

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ABSTRACT: We construct optimum binary spreading sequences of Markov chains in terms of bit error probabilities in asynchronous spread-spectrum multiple-access (SSMA) communication systems based on discretized β-transformations. We also evaluate the normalized auto-correlation function for the optimum binary spreading sequences of Markov chains based on the discretized β-transfor-mations. The experimental results of the bit error probabilities in the asynchronous SSMA communication systems using the obtained sequences agree properly with the theoretical estimations of the bit error probabilities based on the central limit theorem (CLT) in the systems using the optimum spreading sequences of Markov chains.

Multifractal analysis of divergence points of frequency of digits in Bedford-McMullen self-affine carpets

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ABSTRACT: Let K be Bedford-McMullen self-affine carpets generated by IFS $T_{ij}(x,y) = (\frac{x+j}{n}, \frac{y+i}{m})$, where $2 \leq m \leq n$ are integers, and $(i,j) \in D \subseteq \{0,1,\cdots,n-1\} \times \{0,1,\cdots,m-1\}$. We give a detailed analysis of divergence points of frequency of digits in Bedford-McMullen self-affine carpets. More precisely, let $\pi : \Sigma \to K$ be the natural projection, where Σ is the corresponding symbolic space. Denote $\Pi_k(\omega) = (\Pi_k(\omega,d))_{d\in D}$, which gives the frequency of the digits among the first k string in $\omega \in \Sigma$. Let $A(\Pi_k(\omega))$ be the set of the accumulation points of $\Pi_k(\omega)$. The Hausdorff dimensions of the following sets
$\pi(\{\omega \in \Sigma : A(\Pi_k(\omega)) = C\})$ and $\pi(\{\omega \in \Sigma : A(\Pi_k(\omega)) \subseteq C\})$
i.e., the points for which frequency of digits do not exist but behave in a certain prescribed way, are determined for any given subset $C \subseteq \mathbb{R}^{|D|}$.
This is a joint work with Wenxia Li.

The Davies method for heat kernel upper bounds of regular Dirichlet forms

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ABSTRACT: In this talk I will show how to apply the Davies method to obtain upper bounds of heat kernels for regular Dirichlet forms on metric measure spaces, for any walk dimension. Joint with Xuliang Li (Beijing)

Curvature dimension conditions on graphs and related problems

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ABSTRACT: We will introduce curvature dimension conditions on graphs and study some analytic properties for graphs satisfying curvature dimension conditions.

Mobius disjointness for topological models of ergodic systems with discrete spectrum

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ABSTRACT: We provide a criterion for a point satisfying the required disjointness condition in Sarnaks Mobius Disjointness Conjecture. As a direct application, we have that the conjecture holds for any topological model of an ergodic system with discrete spectrum.

Lipschitz equivalence of self-similar sets with exact overlaps

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ABSTRACT: There have been a number of research papers on the Lipschitz equivalence of Cantor sets. Many of those papers focused on dust-like Cantor sets, following the work of Falconer and Marsh. Generally speaking, Lipschitz equivalence of dust-like Cantor sets is often tractable, and in the case of homogeneous IFS the problem is trivial. When the IFS has touching structures, or have different (especially logarithmically non- commensurable contraction ratios) the problem becomes far less trivial. One of the nicest results in this direction is showing the equivalence of the $\{1, 3, 5\}$-Cantor set with the $\{1, 4, 5\}$-Cantor set, where the second IFS has touching structures while the first has none. But when the underlying IFS's have overlaps, most of the techniques we have known so far break down. In this talk, we consider a class of homogeneous self-similar sets with exact overlaps, and give a sufficient condition for the Lipschitz equivalence between members in this class. This is a joint work with Xiu Chen and Wenxia Li.

Randomness criterion – revisited

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ABSTRACT: We propose a randomness criterion $\Sigma^n(x_1x_2\cdots x_n)$ for a binary word $x_1x_2\cdots x_n$, which is the sum of squares of the number of occurrences of every subword of it. The infinite word x satisfying $\lim_{n\to\infty} n^{-2}\Sigma^n(x_1x_2\cdots x_n) = 3/2$ is called a Σ-random word. Almost all words with respect to the $(1/2, 1/2)$-i.i.d. process are Σ-random, and all Σ-random words are normal numbers, but the converse is not true. On the other hand, the Champernowne number is Σ-random. We construct Σ random words and discuss how they are good as random numbers. We also discuss the Σ-value for infinite words with small complexity. This is a joint work with Xue Yu-Mei and Kim Dong Han.

Random walks and induced Dirichlet forms on metric measure spaces

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ABSTRACT: It is known that a self-similar set K can be identified with the hyperbolic boundary $\partial_H X$ of its associated augmented tree $(X, \mathfrak{E})$. In this talk, we will extend the above consideration to compact α-regular metric measure spaces (K, ρ, μ), and study certain reversible random walks with return ratio $\lambda \in (0, 1)$ on $(X, \mathfrak{E})$. We show that the Martin boundary $\mathcal{M}$ can be identified with $\partial_H X$ and K, and the hitting distribution of the walk is exactly the measure μ. With this setup and a device of Silverstein, we are able to attain two-sided estimates of the Martin kernel and the Naim kernel in terms of the Gromov product. Meanwhile, an energy form $\mathcal{E}_K$ on K is induced by the discrete energy $\mathcal{E}_X$ on X, and the Naim kernel turns out to be its jump kernel $\Theta(\xi, \eta) \asymp \rho(\xi, \eta)^{-(\alpha+\beta)}$ where β depends on λ, which implies that the domain of $\mathcal{E}_K$ is a Besov space $\Lambda_{2,2}^{\alpha,\beta/2}$. In order for this $\mathcal{E}_K$ to be a non-local regular Dirichlet form, we further investigate the functional relationship of two energy forms $\mathcal{E}_X$ and $\mathcal{E}_K$, and provide some criteria to determine the critical exponents of Besov spaces $\Lambda_{2,2}^{\alpha,\beta/2}$ through the effective resistances of $\mathcal{E}_X$. By means of network reduction, we calculate the exponents for some concrete examples on self-similar sets. This is a joint work with Ka-Sing Lau and Ting-Kam Leonard Wong.

Critical exponents of Dirichlet forms on self-similar sets

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ABSTRACT: Let $B^{\sigma}_{2,\infty}, \sigma > 0$ denote the Besov space defined on a compact set $K \subset \mathbb{R}^n$ with an α-regular measure μ. The *critical exponent* σ^* is the largest σ such that $B^{\sigma^*}_{2,\infty}$ remains non-trivial. The exponent is determined by the geometry of K and μ. In the analysis of fractals, it is known that for many standard self-similar

sets K, $B^{\sigma^*}_{2,\infty}$ is the domain of some local regular Dirichlet forms, which gives a Laplacian on K. In this talk, we discuss two new examples of p.c.f. self-similar sets that have unusual behavior. This is a joint work with Qingsong Gu.

Hausdorff dimensions of self-affine sets and nonconformal repellers

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ABSTRACT: In fractal geometry and dynamical system, self-similar set and conformal repeller, self-affine set and nonconformal repeller are two pairs of analogous concepts respectively. In the talk, we will compare some results and tools for the Hausdorff dimensions of self-affine sets and nonconformal repellers.

Spectrality of digit sets and spectral self-affine measures

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ABSTRACT: We determine the spectrality of digit set D relating to a spectral self-affine measure $\mu_{M,D}$. This is motivated by a conjecture of Dutkay, Han and Jorgensen. The conjecture states that D is always a spectral set if $\mu_{M,D}$ is a spectral self-affine measure in the dimension $n = 1$. For a self-affine measure $\mu_{M,D}$, we obtain several conditions for the digit set D to be a spectral set. The result here provides some supportive evidence for the conjecture. It also generalizes the corresponding known result in a certain case.

Intersection of a Cantor set with its translation

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ABSTRACT: In this talk we shall consider the intersection of a Cantor set with its translation, its Hausdorff dimension and self-similarity.

A variant of Mandelbrot cascade

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ABSTRACT: Revisiting the founding paper of Kahane and Peyriere on the Mandelbrot multiplicative cascade, we attempt to introduce some kind of variant model linked to other probability subjects. This work is still in progress.

Strict Whitney arcs and t-quasi arcs

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ABSTRACT: The talk is based on joint work with Xin Wei and Zhiying Wen. A connected compact subset E of R^n is said to be a strict Whitney set if there exists a real-valued C^1 function f on R^n with $\nabla f|_E \equiv 0$ such that f is constant on no nonempty relatively open subsets of E. We prove that each nontrivial self-similar arc in R^n is a strict Whitney set. We also study a special kind of self-similar arcs, which we call "regular" self-similar arcs. We obtain necessary and sufficient conditions for a regular self-similar arc Łm to be a t-quasi-arc, and for the Hausdorff measure function on Λ to be a strict Whitney function. We prove that if a regular self-similar arc has "minimal corner angle" $\theta_{\min} > 0$, then it is a 1-quasi-arc and hence its Hausdorff measure function is a strict Whitney function. We provide an example of a one-parameter family of regular self-similar arcs with various features. For some value of the parameter τ, the Hausdorff measure function of the self-similar arc is a strict Whitney function on the arc, and hence the self-similar arc is an s-quasi-arc, where s is the Hausdorff dimension of the arc. For each $t_0 \geq 1$, there is a value of τ such that the corresponding self-similar arc is a t-quasi-arc for each $t > t_0$, but it is not a t_0-quasi-arc. For each $t_0 > 1$, there is a value of τ such that the corresponding self-similar arc is a t_0-quasi-arc, but it is a t-quasi-arc for no $t \in [1, t_0)$.

Assouad dimension of some self-affine fractals

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ABSTRACT: The Assouad dimension was introduced by Assouad in the 1970s, In the talk, we will first review some basic properties of Assouad dimension. Then we will introduce the weak tangent method which is a useful technique to give the lowe bound of Assouad dimension. Finally, we will state some recent results on self-affine sets.

On Random Number generated by Dynamical Systems

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ABSTRACT: : The van der Corput sequences are the most famous low discrepancy sequences. We study these sequences from the view point of dynamical systems.

The discrepancy of sequences depends deeply on the ergodic properties of dynamical systems, especially it depends on the spectra of the PerronCFrobenius operator associated with the dynamical systems.

We studied it by constructing renewal equation of generating functions. In 1Cdimensional cases, the essential spectral radius of the PerronCFrobenius operator equals the reciprocals of the expanding rate, so we can construct general theory. However for higher dimensional cases, the essential spectral radius usually bigger than the reciprocals of the expanding rate. We will construct dynamical systems with their essential spectral radius coincide with the expanding rate using algebraic method.

Maps related to the map

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ABSTRACT: The map(cartgraph) gives a geometrical correspondance between the sphere and the plain, and sometimes suggests sequence of Origami instructions for purpose of carrying the printed paper. We consider several mappings related to the map.

Density of states and level statistics for 1d Schrödinger operators

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ABSTRACT: This is a joint work with Prof. S. Kotani (Osaka Univ.). We consider the 1d Schrödinger operator with random potential decaying of order α. The spectral properties is known to have a transition at $\alpha = 1/2$. The results are:

(1) the fluctuation of density of states with different behavior depending on α,

(2) the level statistics asymptotically obeys clock, Sine β, and Poisson processes for super-critical, critical, and sub-critical cases, respectively.

Spectral asymptotics of one-dimensional fractal Laplacians in the absence of second-order identities

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ABSTRACT: We observe that some self-similar measures defined by finite or infinite iterated function systems with overlaps satisfy certain "bounded measure

type condition", which allows us to extract useful measure-theoretic properties of iterates of the measure. We develop a technique to obtain a closed formula for the spectral dimension of the Laplacian defined by self-similar measures satisfying this condition.

Higher order tangents and higher order Laplacians on symmetric p.d.f. fractals

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ABSTRACT: We study higher order tangents and higher order Laplacians on p.c.f. self-similar sets with fully symmetric structures. The main technical tool is the analysis of local multiharmonic functions and local monomials analogous to $(x - x_0)^j/j!$. The results are closely related to the local Taylor approximations, splines and entire analytic functions. Some of our results can be extended to general p.c.f. fractals.

On the topological classification of fractal squares

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ABSTRACT: A fractal square is a nonempty compact set in the plane satisfying $F = (F+D)/n$, where $n > 1$ is an integer and $D \subset \{0, 1, 2, \cdots, n-1\}^2$ is nonempty. We give the topological classification of fractal squares with $n = 3$ and $\mathrm{Card}(D) = 6$.

Lipschitz equivalence of fractals and finite state automaton

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ABSTRACT: The study of Lipschitz equivalence of fractals is a very active topic in recent years. Most of the studies in literature concern totally disconnected fractals. In this paper, using finite state automata, we construct a bi-Lipschitz map between two fractal squares which are not totally disconnected. This is the first non-trivial map of this type. We also show that this map is measure-preserving.

Some Progresses on the Lipschitz equivalence of self-similar sets

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ABSTRACT: This talk is based on the joint works with Professors Hui Rao, Yang Wang, and Li-Feng Xi. In previous study of Lipschitz equivalence of self-similar

sets, we mainly use algebraical and geometrical method. In this talk, we will present toplogical method to deal with the Lipschitz equivalence of connected self-similar sets.

Inhomogeneous percolation on Bethe lattices

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ABSTRACT: Inhomogeneous percolation, for its closer relationship with real-life, can be more useful and reasonable than homogeneous percolation to illustrate the critical phenomena and dynamical behaviour of complex networks. Due to its intricacy, the theoretical framework of inhomogeneous percolation is far from being complete and many challenging problems are still open. In this paper, we first investigate inhomogeneous site percolation on Bethe lattices with fractal structure and obtain the explicit generalized recursion formulating for the percolation probability and average cluster size and the exact expression for critical occupation probability. Go a step further, we extend the study to an irregular Bethe lattice with random distribution for its sites. The explicit expression for cluster-size distribution of this inhomogeneous site percolation is derived based on probability theory. Moreover, the exact formulas for critical occupation probability, mean cluster size, and percolation probability are obtained by using generating function and generalized recursive approach. In addition, the diffusion behaviour of an infectious disease-SARS in consideration of the two different infection probability is discussed and the specific disease-control strategies is presented.

p-adic continued fractions and Lagrange's theorem

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ABSTRACT: A number of p-adic continued fraction algorithms have been proposed so far, but none of them can generate a periodic continued fraction for every quadratic element of $\mathbb{Q}_p$ over $\mathbb{Q}$. In this talk, we give several continued fraction algorithms, each of which gives an eventually periodic expansion for every quadratic element of $\mathbb{Q}_p$ over $\mathbb{Q}$ and gives a finite expansion for every rational number.

Hausdorff dimension of certain self-affine attractors

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ABSTRACT:
We study the dimension theory of diagonal self-affine IFS and diagonally homogeneous triangular planar self-affine IFS.

Extension of Cantor minimal systems and their dimension groups

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ABSTRACT: In this talk, we consider the relationship between an extension of Cantor minimal systems and order embedding of dimension groups. More precisely, we explain the dynamical realization problem of order embedding of dimension groups.

Topological conditions for the uniqueness of Sinai-Ruelle-Bowen measures

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ABSTRACT: In this talk we consider diffeomorphisms on a closed manifold M preserving a hyperbolic Sinai-Ruelle-Bowen probability measure μ. When the dimension of M is at most three, we give a topological condition which guarantees that there exists at most one hyperbolic ergodic SRB measure. We remark that our condition holds for derived from Anosov (DA) diffeomorphisms on $\mathbb{T}^3$.

p-adic substitutions and Rauzy fractals

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ABSTRACT: We introduce p-adic substitutions, i.e., substitutions over $\mathbb{C}_p$-poweredsymbols. Here, $\mathbb{C}_p$ is the closure of $\mathbb{R}_p$ with respect to the p-adic topology, $\mathbb{R}_p$ is the algebraic closure of $\mathbb{Q}_p$, which is the topological closure of the rational number field $\mathbb{Q}$ with respect to the p-adic topology. Under some convergence results, we can define the Rauzy fractal for a p-adic substitution in the vector space $\mathbb{C}_p^{s+1}$. Making visualization, we show some examples of Rauzy fractals.

On linear independence measures of the values of Mahler functions

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ABSTRACT: We estimate the linear independence measures for the values of a class Mahler functions of degree one and two. For the purpose, we study the determinants of suitable Hermite-Padé approximation polynomials. Based on the non-vanishing of these determinants, we apply the functional equations to get an infinite sequence of approximations which is used to produce the linear independence measures.

Dynamical dimension transference principle for dynamical Diophantine approximation

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ABSTRACT: We aim at finding a general principle for the dynamical Diophantine approximation in a general expanding dynamical system. More precisely, let (X, T) be a topological dynamical system with X a compact metric space and $T : X \to X$ a continuous transformation. Let y_o be a given point in X. Consider the limsup set $\mathcal{W}(T, f)$ driven by the dynamical system (X, T):

$$\left\{x \in X : x \in B(z, e^{-S_n(f+g)(z)}), z \in T^{-n}y_o, \text{ infinitely often } n \in \mathbb{N}\right\}$$

where $g = \log|T'|$, f is a continuous function and $S_n f(z)$ denotes the ergodic sum $f(x) + \cdots + f(T^{n-1}x)$. It is proved that, under some regular conditions on (X, T), both the dimensions of X and $\mathcal{W}(T, f)$ are given respectively by Bowen-Manning-McCluskey formulae, namely the solution to a pressure function:

$$\mathbb{P}(-t\log|T'|) = 0, \quad \mathbb{P}(-t(\log|T'| + f)) = 0.$$

We call this phenomenon the dynamical dimension transference principle as its partially analogy with the mass transference principle in classic Diophantine approximation developed by Beresnevich & Velani.

On the regular sum-free sets

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ABSTRACT: We study the sum-free sets of the Cantor-like sequences and the generalization of the period-doubling sequences. We study the number of zeros (denoted as $\{\mu_n\}_{n\geq 1}$) and stars (denoted as $\{\alpha_n\}_{n\geq 1}$) in the sequence $\{\nu(n)\}_{n\geq 1}$,

and show that these sequences are automatic. Moreover, the difference sequences of the sum-free sets are some morphic sequences. Then, we show the corresponding sum-free sets are regular sum-free sets.

Macroscopic Fractals, Multifractals, and Stochastic Processes

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ABSTRACT: This talk is concerned mainly with the macroscopic fractal and multifractal behavior of various random sets that arise from Brownian motion, Lévy processes, Gaussian processes and solutions to SPDEs. In particular, we provide:

(i) The computation of the macroscopic dimension of the graph of a large family of Lévy processes; and
(ii) Macroscopic multifractal properties of Brownian motion and stochastic heat equations.

This talk is based on joint works with Davar Khoshnevisan (University of Utah) and Kunwoo Kim (POSTECH).

A local generalization of Poincaé-Birkhoff theorem

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ABSTRACT: PoincaréCBirkhoff theorem states that every area and orientation preserving homeomorphism of an annulus that rotates the two boundaries in opposite directions has at least two fixed points. Several generalizations of Poincaré-Birkhoff theorem have been obtained by Carter, Franks, Guillou, Le Calvez, and other authors. We will introduce a local generalization, and as a corollary, give a delicate description of the dynamics of some local homeomorphisms.

On multidimensional p-adic continued fraction

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ABSTRACT: We intoroduce a new multidimensional p-adic continued fraction. Every quadratic element in $\mathbb{Q}_p$ has a periodic continued fraction expansion by this algorithm. Numeric experiments support that every $(\alpha_1, \ldots, \alpha_s)$ where $\{1, \alpha_1, \ldots, \alpha_s\}$ is a $\mathbb{Q}$-base of a field which is included in $\mathbb{Q}_p$ will have a periodic continued fraction expansion by this one.

On the digits distribution of β and $(-\beta)$ expansions

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ABSTRACT: Let $T_\beta(x) = \beta x - \lfloor \beta x \rfloor$ and $T_{-\beta}(x) = -\beta x + \lfloor \beta x \rfloor + 1$ be the beta and negative beta transformations respectively. The two maps induce β-expansions and $(-\beta)$-expansions respectively. The asymptotic estimate of the length of consecutive 0 digits are studied for β-expansions. The relationship of the $(-\beta)$-expansions of 1 and some topological properties of $T_{-\beta}$ are also investigated. This is a joint work with Hui Hu, Xin Tong and Yanfen Zhao.

Odometer Representation of 3-Interval Exchange, revisited

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ABSTRACT: Y. Ichikawa and S. Ito (2009) have constructed an odometer (or adic) representation of a non-degenerate 3-interval exchange transformation, via the Nakazawa-Ito-Rao algorithm. From the viewpoint of the framework of ordered Bratteli diagrams, their ordered Bratteli diagram is not properly ordered in general. Here, we say that an ordered Bratteli diagram is properly ordered if it has the unique minimal path and the unique maximal path. In this talk, a modified Nakazawa-Ito-Rao algorithm is proposed, and via this modified algorithm, an odometer representation with a properly ordered Bratteli diagram can be constructed.

Fractal percolation is unrectifiable

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ABSTRACT: We show that there exists $0 < \alpha_0 < 1$ (depending on the parameters) such that the fractal percolation is almost surely purely α-unrectifiable for all $\alpha > \alpha_0$.

Asymptotic quantization error for Markov-type measures

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ABSTRACT: We study the asymptotic quantization error for Markov-type measures μ on a class of ratio-specified graph directed fractals. We show that the quantization dimension for μ of order r exists and determine its exact value s_r in terms of the spectral radius of a related matrix. We establish a necessary and sufficient condition for the s_r-dimensional upper and lower quantization coefficient for μ to be positive and finite.

A proof of the conjecture on twin primes

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ABSTRACT: Denote P be the set of all prime numbers: $P = \{p_1, p_2, p_3, p_4 \cdots, \}$, where $p_1 = 1, p_2 = 2, p_3 = 3, p_4 = 5, \cdots$. Denote P_1 the set of positive integers. In P_1, cross out all multiples of 2 greater than 2, and the set remaining denoted by P_2. In P_2 cross out all multiples of 3 greater than 3, and the set remaining. Inductively, in P_{i-1} cross out all multiples of the next prime P_{i-1} greater than $Pi - 1$ and the set remaining denoted by P_i. So we obtain that $P_1 \supset P_2 \supset P_3 \supset \cdots$. It is easy to see that

$$P = \cup_{i=1}^{\infty} P_i.$$

To form a subsequence of positive integers as follows: Let $n_j = p_3 p_4 \cdots p_j + 2, n_j + 2 = p_3 p_4 \cdots p_j + 4, j > 3$. It is easy to see that $n_j, n_j + 2 \in P_i, \forall i \geq 1$. Denote $P_{i,n}$ the n-th element in $P_i, \forall n > 0$. It is easy to see that

$$\liminf_{n\to\infty}(p_{n+1} - p_n) = \liminf_{n\to\infty} \lim_{i\to\infty}(p_{p,n+1} - p_{p,n}).$$

For $j \geq l$, let n_j be the $m_{j,i}$-th element in P_i, that is, $p_{l,m_{i,j}} =_j$, and clearly, $p_{i,m_{i,j}+1} = n_j + 2$. Consider the limit $\liminf_{n\to\infty}(p_{i,n+1} - p_{i,n})$. For given $l > 2, \{m_{j,i}, mj, i{+}1\}_{j\geq l}$ is a subsequence of positive integers and using the subsequence, we have

$$\liminf_{n\to\infty}(p_{n+1} - p_n) \leq \lim_{j\to\infty}(p_{i,m_{i,j}+1} - p_{i,m_{i,j}}) = n_j + 2 - n_j = 2.$$

So, we have that

$$\{p_{4,m_{i,4}+1} - p_{4,m_{i,4}}, p_{5,m_{i,5}+1} - p_{5,m_{i,5}}, \cdots, p_{j,m_{i,j}+1} - p_{j,m_{i,j}}, \cdots\} = \{2, 2, \cdots\},$$

is a constant value sequence of 2, therefore we have

$$2 \leq \liminf_{n\to\infty}(p_{n+1} - p_n) = \liminf_{n\to\infty} \lim_{i\to\infty}(p_{i,m_{i,j}+1} - p_{i,m_{i,j}}) = 2$$

We are done.

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