The 16th International Conference on Representations of Algebras and Workshop

Tsinghua Sanya International Mathematics Forum (TSIMF) Sanya, Hainan Province, China August 20–29, 2014

Abstracts

Workshop Program, ICRA 2014 , August 20-23, 2014 (TSIMF, Main Lecture Hall)

Date	August 20	August 21	August 22	August 23
Time				
09:00-09:50	Registration	Emmanuel Letellier	Geordie Williamson	Jan Schröer
		Lecture 1	Lecture 3	Lecture 4
10:00-10:50	Claus Michael Ringel	Jan Schröer	Jan Schröer	Graham Leuschke
	Lecture 1	Lecture 2	Lecture 3	Lecture 4
10:50-11:20	Tea Break	Tea Break	Tea Break	Tea Break
11:20-12:10	Graham Leuschke	Geordie Williamson	Claus Michael Ringel	Emmanuel Letellier
	Lecture 1	Lecture 2	Lecture 4	Lecture 4
12:10-14:00	Lunch break	Lunch break	Lunch break	Lunch break
14:00-14:50	Geordie Williamson	Claus Michael Ringel	Graham Leuschke	Error
	Lecture 1	Lecture 3	Lecture 3	rree
14:50-15:20	Tea Break	Tea Break	Tea Break	Afternoon
15:20-16:10	Claus Michael Ringel	Emmanuel Letellier	Emmanuel Letellier	-
	Lecture 2	Lecture 2	Lecture 3	
16:20-17:10	Jan Schröer	Graham Leuschke	Geordie Williamson	_
	Lecture 1	Lecture 2	Lecture 4	
17:45-	Dinner	Dinner	Dinner	

Workshop Abstracts

Emmanuel Letellier (University of Caen)

Kac polynomials, DT-invariants, representations of $GL_n(q)$

In this lecture we will use the geometric representation theory of $GL_n(q)$ to investigate certain quantities such as Kac polynomials, DT-invariants and the counting of parabolic bundles over the projective line. In particular we will give the proof for positivity of Kac polynomials and DT-invariants and we will also give results and precise conjectures for the counting of geometrically indecomposable parabolic structures on a given vector bundle over the projective line.

Graham Leuschke (Syracuse University)

Non-commutative desingularizations and Cohen-Macaulay representations

These lectures will describe the work done over the last several years, by many sets of authors, on non-commutative analogues of resolutions of singularities. I will start by discussing the McKay Correspondence, a key motivating example, and then consider several desiderata in determining what a non-commutative desingularization should be, including Van den Bergh's definition of non-commutative crepant resolutions. Throughout I will emphasize the role played by maximal Cohen-Macaulay modules.

Claus Michael Ringel (Bielefeld University)

The root posets

A root system is a finite set of vectors in a Euclidean vector space satisfying some strong symmetry conditions. Root systems are used as convenient index sets when dealing with semi-simple complex Lie algebras or algebraic groups, but play an important role also in other parts of mathematics. The root systems have been classified by Killing and Cartan at the end of the 19th century, the different types of irreducible root systems are labeled by the Dynkin diagrams A_n, B_n, \ldots, G_2 . As we mentioned, the definition of the root systems refers to symmetry properties, but it turns out that there are further hidden symmetries which are not at all apparent at first sight. They have been discovered only quite recently and extend the use of root systems considerably.

If A is a hereditary artin algebra of finite representation type, it is well-known that the indecomposable A-modules correspond bijectively to the positive roots of a root system. The positive roots form in a natural way a poset, these posets are called the root posets. In the setting of A-modules, the ordering is given by looking at subfactors. Root posets play a decisive role in many parts of mathematics: of course in Lie theory, in geometry (hyperplane arrangements) and group theory (reflection groups), but also say in singularity theory, in topology, and even in free probability theory (non-crossing partitions). The aim of the lectures will be to report on combinatorial properties of the root posets which have been found in recent years by various mathematicians, in view of these applications. Of course, whenever possible, we will focus the attention to the relevance of these properties in the representation theory of hereditary artin algebras.

Several of the results which we will discuss have been generalized to the Kac-Moody root systems, but we usually will restrict to the finite root systems.

Jan Schröer (University of Bonn)

Algebras associated to Cartan matrices

There are numerous (associative) algebras associated to (symmetrizable generalized) Cartan matrices. Their representation theory and the root system of an associated Kac-Moody Lie algebra are intimately linked. I will review some of the classical theory, mainly focussing on symmetric Cartan matrices. Then I will discuss some new approach to the symmetrizable case, which is part of recent joint work with Christof Geiss and Bernard Leclerc.

Geordie Williamson (Max-Planck Institute)

Representations of algebraic groups, Lusztig's conjecture and Soergel calculus

Finding the irreducible rational representations of algebraic groups is a basic question in representation theory. I will start with an introduction to the subject, recalling the classification via highest weight, the Frobenius twist, the Steinbert tensor product theorem and the linkage principle. I will then move to more modern developments, and discuss the work of Andersen-Jantzen-Soergel, Soergel and Fiebig which gives combinatorial models for these questions. Finally I will discuss Soergel calculus (generators and relations for Soergel bimodules) and explain why the situation is much more complicated than we might have thought. Parts of the new work I will discuss is joint with Ben Elias and Simon Riche.

Conference Abstracts

Takahide Adachi (Nagoya University) *Tilting-discrete Brauer graph algebras*

A symmetric algebra is called *tilting-discrete* if, for any positive integer l, there exist only finitely many tilting complexes with homologies concentrated only in degrees $0, 1, \dots, l$. For example, every representation-finite symmetric algebra is tilting-discrete. In a joint work with Takuma Aihara and Aaron Chan, we study the tilting-discreteness of Brauer graph algebras, which are representation-tame symmetric algebras defined by locally embedded graphs. We prove that for a Brauer graph algebra, it is tilting-discrete if, and only if its associated graph is a tree, or has only one subcycle of odd length and none of even length. In this talk, I will introduce the notions needed and explain the motivation. If time allows, I will briefly explain the techniques we used.

Tokuji Araya (Okayama University of Science) Gorensteinness on the punctured spectrum

This is a joint work with K. Iima.

Let R be a commutative noetherian ring. A finitely generated R-module C is called *semidualizing* if the homothety map $R \to \operatorname{Hom}_R(C, C)$ is an isomorphism and if $\operatorname{Ext}_R^{>0}(C, C) = 0$. A free module of rank one and a dualizing module are semidualizing modules. A notion of *n*-torsionfree has been introduced by Auslander and Bridger as generalization of reflexive.

In this talk, we will characterize *n*-torsionfreeness of modules with respect to a semidualizing module in terms of the Serre's condition (S_n) . As an application we will give a characterization of Cohen-Macaulay rings which is Gorenstein on the punctured spectrum.

Javad Asadollahi (University of Isfahan and IPM) Derived equivalence versus Gorenstein derived equivalence

We study Gorenstein derived categories from different points of view, study the relationships between Gorenstein and (absolute) derived categories, specially over artin algebras, and provide some invariants under Gorenstein derived equivalences. The talk is based on a joint work with Razieh Vahed and Rasool Hafezi.

Hideto Asashiba (Shizuoka University)

Gluing derived equivalences together with bimodules

Throughout this talk we fix a commutative ring k and a small category I with I_0 (resp. I_1) the class of objects (resp. morphisms). Consider the bicategory k-Cat^b of all small k-categories whose 1-morphisms are the bimodules over them (1-morphisms from \mathcal{C} to \mathcal{D} are the \mathcal{D} - \mathcal{C} -bimodules ${}_{\mathcal{D}}M_{\mathcal{C}}$ and the composite $\mathcal{C} \to \mathcal{D} \to \mathcal{E}$ is given by the tensor product over \mathcal{D}) and whose 2-morphisms are the bimodule morphisms. We can define a 2-category structure on the class of lax functors $I \to \mathbb{k}\text{-}\operatorname{Cat}^b$. The obtained 2-category is denoted by $\overrightarrow{\text{Lax}}(I, \Bbbk\text{-Cat}^b)$. We define (a generalized version of) the Grothendieck construction Gr(X) of a lax functor $X: I \to \Bbbk$ -Cat^b, which enables us to construct new k-categories by tying k-categories X(i) ($i \in I_0$) together with bimodules $X(a)(a \in I_1)$. In particular, this construction can present a triangular matrix algebra of the form $\begin{pmatrix} A & 0 \\ M & B \end{pmatrix}$ with A, B k-algebras and M a B-A-bimodule, or more generally the tensor algebra of a k-species. For a lax functor $X: I \to k\text{-}Cat^b$, we define its "module category" ModX and its "derived module category" $\mathcal{D}(ModX)$, both of which are again lax functors from I. We also define a notion of derived equivalences between lax functors $I \to \Bbbk$ -Cat^b and a notion of tilting lax subfunctors of the lax functor $\mathcal{D}(ModX)$, which is a generalization of tilting subcategory of the category $\mathcal{D}(Mod\mathcal{C})$ for a k-category \mathcal{C} .

When k is a field, we can construct a derived equivalence between the Grothendieck constructions Gr(X) and Gr(X') of lax functors $X, X' : I \to k$ -Cat^b by gluing derived equivalences between X(i) and $X'(i)(i \in I_0)$ together with bimodules X(a)and $X'(a)(a \in I_1)$ if X and X' are derived equivalent. More precisely we have the following.

Theorem. Assume that \Bbbk is a field. Let X, X' be lax functors $I \to \Bbbk$ -Cat^b and consider the following conditions:

- (a) X and X' are derived equivalent;
- (b) X' is equivalent to a tilting lax subfunctor \mathcal{T} of $\mathcal{D}(ModX)$ in the 2-category $\overrightarrow{Lax}(I, \Bbbk-Cat^b)$; and
- (c) Gr(X) and Gr(X') are derived equivalent as k-categories.

Then (a) implies (b), and (b) implies (c).

In particular, the theorem above gives us the following corollary that constructs derived equivalent pairs of k-algebras using tilting complexes and bimodules.

Corollary. Let A_1, A_2 be algebras over a field \Bbbk and ${}_{A_2}M_{A_1}$ a A_2 - A_1 -bimodule. For each i = 1, 2 take a tilting complex $T_i \in \mathcal{K}^b(\operatorname{prj} A_i)$ and set $A'_i := \mathcal{K}^b(\operatorname{prj} A_i)(T_i, T_i)$, $M' := \mathcal{D}(\operatorname{Mod} A_1)(T_1, T_2 \overset{\mathbf{L}}{\otimes}_{A_2} M)$. Assume that $\mathcal{D}(\operatorname{Mod} A_1)(T_1, T_2 \overset{\mathbf{L}}{\otimes}_{A_2} M[n]) = 0$ for all integers $n \neq 0$. Then $\begin{pmatrix} A_1 & 0 \\ M & A_2 \end{pmatrix}$ and $\begin{pmatrix} A'_1 & 0 \\ M' & A'_2 \end{pmatrix}$ are derived equivalent.

Jerzy Białkowski (Nicolaus Copernicus University) Periodic algebras of polynomial growth

This is report on joint work with K. Erdmann and A. Skowroński.

Let A be a finite dimensional K-algebra over an algebraically closed field K. Denote by Ω_A the syzygy operator on the category modA of finite dimensional right Amodules, which assigns to a module M in modA the kernel $\Omega_A(M)$ of a minimal projective cover $P_A(M) \to M$ of M in modA. A module M in modA is said to be periodic if $\Omega_A^n(M) \cong M$ for some $n \ge 1$. Then A is said to be a periodic algebra if A is periodic in the module category modA^e of the enveloping algebra $A^e = A^{op} \otimes_K A$, that is, periodic as an A-A-bimodule. The periodic algebras A are selfinjective and their module categories modA are periodic (all modules in modA without projective direct summands are periodic). The periodicity of an algebra A is related with the periodicity of its Hochschild cohomology algebra $HH^*(A)$ and is invariant under equivalences of the derived category $D^b(\text{mod}A)$ of bounded complexes over modA. One of the exciting open problems in the representation theory of selfinjective algebras is to determine the Morita equivalence classes of periodic algebras.

It has been proved by Dugas that every selfinjective algebra of finite representation type, without semisimple summands, is a periodic algebra. During the talk we will present a description of all basic, indecomposable, representation-infinite periodic algebras of polynomial growth.

Grzegorz Bobinski (Nicolaus Copernicus University) Moduli spaces for quasi-titled algebras

There are ongoing efforts to characterize the representation type of an algebra in terms of the associated King's moduli spaces. I will discuss the problem for the quasititled algebras. In particular, I will show that a quasi-tilted algebra is tame if and only if the corresponding moduli spaces are products of projective spaces.

Thomas Brüstle (Bishop's University and Université de Sherbrooke) On the boundary algebra of a marked surface

At the ICRA 2007 in Torun, I discussed properties of the Jacobian algebra defined by the internal arcs of a triangulation σ of a marked surface (S, M). I always wanted to include the boundary arcs into the picture, but only recently progress was made by Baur, King and Marsh [1] when (S, M) is an unpunctured disc, and independently by Demonet and Luo when (S, M) is an unpunctured disc [2] and when (S, M) a disc with one puncture [3].

Their strategy is to define an algebra $\Gamma(\sigma)$ containing the boundary arcs as "frozen" vertices, and to study the "boundary algebra" $B(S, M) = e\Gamma(\sigma)e$ where e is the

idempotent of $\Gamma(\sigma)$ formed by all boundary arcs. As the notation suggests, one can show that the algebra B(S, M) depends (up to isomorphism) only on the marked surface (S, M) and not on the chosen triangulation σ . On the other hand, each triangulation σ defines a maximal Cohen-Macaulay *B*-module $\Gamma(\sigma)e$ without selfextensions.

The independence from the triangulation is beautiful, but one has to pay a price for it: the algebra B is no longer finite-dimensional. However, in the cases discussed in [1, 2, 3], the authors show that B is an order over the polynomial ring k[t], in fact they show B is isomorphic to the skew group algebra $S * \mathbb{Z}_n$ for a simple singularity S, and hence (and as it is shown independently), B is a Gorenstein ring. Consequently, the stable category $\underline{CM}(B)$ of Cohen-Macauly modules is triangulated, it admits the modules $\Gamma(\sigma)e$ as (cluster-)tilting objects, and in fact $\underline{CM}(B)$ is equivalent to the cluster category of type A when (S, M) is an unpunctured disc [1, 2], and of type Dwhen (S, M) a disc with one puncture [3].

Things are getting rapidly more complicated when studying the case of general marked surfaces, I plan to mention the few results that I can show in general, and point out what are the obstacles at this moment.

References

- K. Baur, A. King and R. J. Marsh, Dimer models and cluster categories of Grassmannians, arXiv: 1309.6524
- [2] L. Demonet and X. Luo, Ice quivers with potentials associated with triangulations and Cohen-Macaulay modules over orders, arXiv: 1307.0676
- [3] L. Demonet and X. Luo, Ice quivers with potential arising from once-punctured polygons and Cohen-Macaulay modules, arXiv: 1404.7269

Ragnar-Olaf Buchweitz (University of Toronto)

Higher representation-infinite algebras from geometric tilting objects

We want to report on joint work with Lutz Hille, Universität Münster, Germany, on the recent notion of higher representation-infinite algebras. We show that a tilting object in a triangulated category of geometric dimension d, a notion proposed by Bondal, has an endomorphism ring that is higher representation-infinite if, and only if, it pulls back to a tilting object on the virtual affine canonical bundle over that category if, and only if, the endomorphism algebra has minimal global dimension, equal to d, and the tilting object is sheaf-like.

The endomorphism ring of the pullback then yields the corresponding higher preprojective algebra. This proves, for example, that any full cyclic strongly exceptional sequence, a notion due to Hille-Perling that comprises the classical notion of a helix, gives rise to such pairs of *d*-representation-infinite algebras and their accompanying higher (d + 1)-preprojective algebras, thereby providing plenty of examples. Intriguingly, such algebras also arise on non-Fano varieties, such as the second Hirzebruch surface or some non-isolated quotient singularities defined by abelian subgroups of special linear groups.

Modulo an outstanding conjecture on the graded coherence of higher preprojective algebras and results of Minamoto, it follows that representation-infinite algebras are precisely the ones arising as endomorphism rings of minimal global dimension of sheaf-like tilting objects in triangulated categories of geometric dimension d.

Jon F. Carlson (University of Georgia)

The group of torsion endotrivial modules

Let G be a finite group and k a field of characteristic p, dividing the order of G. A kG-module is endotrivial if its endomorphism ring $\operatorname{Hom}_k(M, M)$, as a kG-module, is isomorphic to k in the stable category, $\operatorname{stmod}(kG)$. Tensoring with an endotrivial module is a self-equivalence of Morita type on $\operatorname{stmod}(kG)$. The endotrivial module have been classified in the case that G is a p-group and in many other cases. Recent new techniques suggested by work of Balmer, have greatly enhanced the machinery for such characterizations. It is now possible to give a complete and elementary local description of the group of endotrival modules in the case that the Sylow p-subgroup of G is abelian. The talk with will highly joint work with Jacques Thevenaz, Nadia Mazza and Dan Nakano.

Giovanni Cerulli Irelli (Sapienza-UNiversità di Roma) Quivers with self-duality and isotropic quiver Grassmannians

We consider a finite dimensional algebra A = kQ/I given by a quiver with relations, with the extra property that the quiver Q is endowed with an involutive anti-automorphism which leaves the ideal I invariant. Such an antiautomorphism induces an involutive self-duality on the cateogry modA of finite dimensional Amodules. Motivated by problems in linear algebra, like classification problems of orthogonal-symplectic multiple flag varieties, I will present some results concerning self-dual representations. In particular, I will consider the subrepresentations which are isotropic and I will provide some geometric properties.

Jianmin Chen (Xiamen University)

Classification of tilting objects on a weighted projective line of type $(2, 2, 2, 2; \lambda)$

We investigate the stable category of vector bundles on a weighted projective line of type $(2, 2, 2, 2; \lambda)$ obtained from the vector bundles by factoring out all the line bundles. We show that each tilting object in the stable category pushes to a cluster tilting object in its cluster category, and describe all the tilting objects corresponding to a given cluster tilting object in the cluster category. Moreover, we realize the classifications of all the endomorphism algebras of the tilting objects in the category of coherent sheaves and in its derived category.

This is joint work with Yanan Lin, Pin Liu and Shiquan Ruan.

Raquel Coelho Simoes (Universidade de Lisboa)

Torsion pairs in a triangulated category generated by a spherical object

There has been intense recent interest in the triangulated categories T_w , which are each generated by a *w*-spherical object. These provide a family of categories which are *w*-Calabi-Yau. Moreover, when $w \ge 2$, T_w can be seen as a *w*-cluster category of type A infinity.

In this talk we will give a characterisation of torsion pairs in T_w , for arbitrary w, extending Ng's characterisation for T_2 in terms of Ptolemy arcs. We will see that when w is negative, there is a surprisingly pleasant combinatorial description in terms of *modified Ptolemy arcs*. This is joint work with David Pauksztello (Manchester).

William Crawley-Boevey (University of Leeds) *Two applications of the functorial filtration method*

The method in question is the one used by Gelfand and Ponomarev in the classification of annihilating operators (to which they reduced the classification of certain indecomposable representations of the Lorentz group). It was further developed by Gabriel, and used by Ringel to classify indecomposable representations of dihedral groups.

In the functorial filtration method one shows that a candidate list of indecomposable representations is complete by writing down two explicit vector space filtrations of a representation and verifying certain compatibilities between the list and the filtrations.

I will discuss the use of this method in the following two cases:

- (1) the classification of finitely generated modules for infinite dimensional string algebras such as k[x, y]/(xy), and
- (2) the classification of persistence modules, that is, functors from a totally ordered set, considered in the natural way as a category, to the category of vector spaces. Such modules arise in the study of persistent homology.

Darmajid Darmajid (Institut Teknologi Bandung)

Varieties of complexes projective of fixed rank

This is joint work with Bernt Tore Jensen. We study varieties of complexes of projective modules with fixed ranks, and relate these varieties to the varieties of their homologies. We show that for an algebra of global dimension two, these two varieties are related by a pair of morphisms which are smooth with irreducible fibres.

Erik Darpö (Malardalen University) *n*-representation-finite self-injective algebras

In the classical Auslander-Reiten theory, there is a strong link between representationfinite hereditary and self-injective algebras. In this talk, I shall explain some recent research by Iyama and myself, seeking to generalise this connection from the point of view of higher-dimensional Auslander-Reiten theory.

I will introduce self-injective *n*-representation-finite algebras and present a method, generalising the classical one, using which all known instances of such algebras can be constructed. The main tool is a result showing that *n*-cluster-tilting subcategories are, in a nice way, preserved by Galois coverings.

José-Antonio de la Peña (UNAM and CIMAT, Guanajuato, México) Mahler measure of Coxeter matrices and the representation type of an algebra

By a celebrated result of Kronecker, a monic integral polynomial \mathbf{p} (with $\mathbf{p}(0) \neq 0$) has Mahler measure $\mathbf{M}(\mathbf{p})=1$ if and only if \mathbf{p} factorizes as product of cyclotomic polynomials. Rather little is known, however, about values of the Mahler measure near 1. In 1933, D. H. Lehmer found that the polynomial $T^{10} + T^9 - T^7 - T^6 - T^5 - T^4 - T^3 + T + 1$ has Mahler measure $\mu_0 = 1.176280...$, and he asked if there exist any smaller values exceeding 1. In fact, the polynomial above is the Coxeter polynomial of the hereditary algebra with underlying graph of tree type [2,3,7]. This is still an open conjecture.

We explore the topic of the Mahler measure $\mathsf{M}(\chi_A)$ of the Coxeter polynomial χ_A of triangular algebras A. We say that an algebra A is of *cyclotomic type* if $\mathsf{M}(\chi_A) = 1$. There are many important examples of algebras of cyclotomic type: hereditary tame algebras, canonical algebras, among others. On the other hand, there are simple examples of representation-finite algebras B with $\mathsf{M}(\chi_B) > 1$. Some results:

Theorem 1: Let A be a strongly simply connected algebra not of cyclotomic type. There exists a convex subcategory C of A such that $M(\chi_C) \ge \mu_0$. **Theorem 2**: Let A be a strongly simply connected algebra. Then A is of tame representation type if and only if any convex subcategory B of A is either representation-finite or of cyclotomic type.

Corollary: Let A be a strongly simply connected algebra. Suppose for every convex subcategory B of A with at most 10 vertices is representation-finite or has $M(\chi_B) = 1$ then A is tame.

We describe the structure of the derived category of A using the cyclotomic factors of χ_A . With the use of Galois coverings we extend the above Theorems to further classes of algebras.

Laurent Demonet (Nagoya University)

From categories of Cohen-Macaulay over orders to subcategories of modules categories

This is a report about joint works with Xueyu Luo and works in progress with Osamu Iyama.

Recently, several papers [BKM, DLb, DLa, JKS] introduced Gorenstein orders Λ over R = k[[x]] such that the stable category <u>CM</u> Λ of Cohen-Macaulay Λ -modules is triangle equivalent to a cluster category. In all these cases, it turns out that there is an idempotent e of Λ such that $A = \Lambda/e\Lambda e$ is a finite-dimensional algebra over k and there is an equivalence of categories CM $\Lambda \cong$ SubA where SubA is the full subcategory of modA consisting of the submodules of A^n for any $n \in \mathbb{N}$. The aim of this talk is to discuss a general framework extending these results. We give a general result concerning any order and idempotent such that $\Lambda/e\Lambda e$ is finite dimensional. Most of the times it only gives well behaved functor between subcategories of CM Λ and SubA. Assuming stronger assumptions, it gives an equivalence of categories. We finally concentrate to the case where A is a quotient of a preprojective algebra of Dynkin type defined by an element of the Weyl group (as in previously studied cases, see [BIRS09] for more details about these quotients). We give several examples (including and generalizing the ones of [BKM, DLb, DLa, JKS]) and discuss obstacles for certain other algebras.

References

- [BIRS09] A. B. Buan, O. Iyama, I. Reiten, and J. Scott. Cluster structures for 2-Calabi-Yau categories and unipotent groups. *Compos. Math.*, 145(4):1035-1079, 2009.
- [BKM] Karin Baur, Alastair King, and Robert J. Marsh. Dimer models and cluster categories of Grassmannians. arXiv: 1309.6524.
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[JKS] Bernt Jensen, Alastair King, and Xiuping Su. A category for Grassmannian cluster algebras. arXiv: 1309.7301.

Y. A. Drozd (National Academy of Sciences of Ukraine) Categorical resolutions of singular non-commutative curves

It is a joint work with I. Burban and V. Gavran, see [1].

A projective non-commutative curve over an algebraically closed field \mathbf{k} is a pair (X, \mathcal{A}) , where X is a projective curve over \mathbf{k} and \mathcal{A} is a sheaf of $\mathcal{O}_{\mathcal{X}}$ -algebras which is coherent torsion free as a sheaf of $\mathcal{O}_{\mathcal{X}}$ -modules and contains no nilpotent ideals. Then there is a chain of over-rings of $\mathcal{A} : \mathcal{A} = \mathcal{A}_0 \subset \mathcal{A}_1 \subset \mathcal{A}_2 \subset \cdots \subset \mathcal{A}_n \subset \mathcal{A}_{n+1} = \tilde{\mathcal{A}}$, where $\tilde{\mathcal{A}}$ is hereditary and if $\mathcal{J}_k = \mathcal{H}om_{\mathcal{A}}(\mathcal{A}_{i+1}, \mathcal{A}_i)$ considered as an ideal in \mathcal{A}_i , then $\mathcal{A}_i/\mathcal{J}_i$ is semi-simple. Set $\mathcal{A}_{ij} = \mathcal{J}_j\mathcal{J}_{j+1}\cdots\mathcal{J}_{i-1}$ if i > j and $\mathcal{A}_{ij} = \mathcal{A}_j$ if $i \leq j$. We define the König's resolution of \mathcal{A} as the sheaf of algebras \mathcal{R} consisting of $(n+1) \times (n+1)$ matrices (a_{ij}) , where $a_{ij} \in \mathcal{A}_{ij}$. It is known that every localization \mathcal{R}_x $(x \in X)$ is a quasi-hereditary order [2]. Let e_i be the diagonal matrix units, $\mathcal{P} = \mathcal{R}e_1, \tilde{\mathcal{P}} = \mathcal{R}e_{n+1}, \mathcal{I} = \mathcal{A}e_{n+1}\mathcal{A}$ and $\mathcal{Q} = \mathcal{R}/\mathcal{I}$. We identify \mathcal{Q} with the finite dimensional algebra of its global sections. It is a quasi-hereditary algebra. We consider the functors

$$\begin{split} \mathrm{F} &= \mathcal{P} \otimes_{\mathcal{A}} - : \mathrm{Qcoh}\mathcal{A} \to \mathrm{Qcoh}\mathcal{R}, \mathrm{G} = \mathcal{H}\mathrm{om}_{\mathcal{R}}(\mathcal{P}, -) : \mathrm{Qcoh}\mathcal{R} \to \mathrm{Qcoh}\mathcal{A}, \\ \tilde{\mathrm{F}} &= \mathcal{P} \otimes_{\tilde{\mathcal{A}}} - : \mathrm{Qcoh}\tilde{\mathcal{A}} \to \mathrm{Qcoh}\mathcal{R}, \\ \tilde{\mathrm{G}} &= \mathcal{H}\mathrm{om}_{\mathcal{R}}(\mathcal{P}, -) : \mathrm{Qcoh}\mathcal{R} \to \mathrm{Qcoh}\tilde{\mathcal{A}}. \end{split}$$

Theorem 1

- 1. G is a localizing functor, so $\operatorname{Qcoh}\mathcal{A} \simeq \operatorname{Qcoh}\mathcal{R}/\operatorname{ker}G$ and F is fully faithful.
- 2. $\mathcal{D}(\operatorname{Qcoh}\mathcal{R}) = \langle \operatorname{ker}\tilde{G}, \operatorname{im}\tilde{F} \rangle$ is a semi-orthogonal decomposition of the derived category $\mathcal{D}(\operatorname{Qcoh}\mathcal{R})$.
- 3. ker $\hat{\mathbf{G}} \simeq \operatorname{Qcoh} \mathcal{Q}$ and im $\hat{\mathbf{F}} \simeq \operatorname{Qcoh} \hat{\mathcal{A}}$.
- 4. dim $\mathcal{D}^{b}(\operatorname{Coh}\mathcal{A}) \leq n + \operatorname{dim}\mathcal{D}^{b}(\operatorname{Coh}\tilde{\mathcal{A}})$. In particular, if X is rational, then dim $\mathcal{D}^{b}(\operatorname{Coh}\mathcal{A}) \leq n+2$ and if, moreover, the curve is commutative, *i.e.* $\mathcal{A} = \mathcal{O}_{X}$, then dim $\mathcal{D}^{b}(\operatorname{Coh}\mathcal{A}) \leq n+1$.

The corresponding strong generators for $\mathcal{D}^b(\mathrm{Coh}\mathcal{A})$ are described explicitly.

References

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Florian Eisele (Vrije Universiteit Brussel) Lifting group rings and tame blocks

I will talk about the problem of lifting a finite-dimensional algebra $\overline{\Lambda}$ defined over a field F to an order Λ defined over a discrete valuation ring \mathcal{O} with residue field F. In this context, Λ being a "lift" of $\overline{\Lambda}$ means that $F \otimes \Lambda \cong \overline{\Lambda}$. Specifically, I am interested in the case where $\overline{\Lambda}$ is the basic algebra of a block of a group ring of a finite group. The problem then is to find the basic algebra of the corresponding block over \mathcal{O} . I will present an approach to this problem which exploits derived equivalences between different blocks. Using this approach I was able to obtain explicit descriptions for several classes of tame blocks defined over a discrete valuation ring with residue field $\overline{\mathbb{F}}_2$ (starting from the classification of these blocks over $\overline{\mathbb{F}}_2$). In the case of blocks of dihedral defect with two simple modules this helped narrow down which Morita equivalence classes in the classification of tame blocks can actually occur in group rings. Moreover, by using that very same approach, I was able to prove that a conjectural description of the basic algebra of $\mathbb{Z}_p[\zeta_{p^f-1}]SL_2(p^f)$ is indeed correct.

Eleonore Faber (University of Toronto) Noncommutative resolutions and the global spectrum of a commutative ring

Motivated by algebraic geometry, one studies non-commutative analogs of resolutions of singularities. In short, non-commutative resolutions of commutative rings Rare endomorphism rings of certain R-modules of finite global dimension. However, it is not clear which values of finite global dimensions are possible, even for rings of low Krull-dimension. This leads us to consider the so-called global spectrum of a ring, that is $gs_{MCM(R)}(R)$, the set of all possible global dimensions of endomorphism rings of Cohen-Macaulay-modules. When R is artinian and X is the category of modules which are both generator and cogenerator, then the minimum value in $gs_X(R)$ has been studied as representation dimension of R (initiated by M. Auslander).

In this talk we will consider non-commutative resolutions for non-normal rings, in particular the question, under which conditions a non-commutative resolution exists. Then we will address some questions connected with the global spectrum and discuss several examples coming from algebraic geometry. This is joint work with H. Dao and C. Ingalls.

Jiarui Fei (University of California, Riverside) Cluster algebras and semi-invariant rings

There are two well-known models for studying (type-A) Littlewood-Richardson coefficients. One is Knutson-Tao's hive model [7], the other is Derksen-Weyman's

approach via quiver semi-invariants [2]. The quiver representation space involved is the triple flag quiver T_n with dimension vector β as indicated below:

$$1 \longrightarrow 2 \longrightarrow \cdots \longrightarrow n-1$$

$$1 \longrightarrow 2 \longrightarrow \cdots \longrightarrow n-1 \longrightarrow n$$

$$1 \longrightarrow 2 \longrightarrow \cdots \longrightarrow n-1$$

We find quite amazingly that each such a semi-invariant ring contains a cluster algebra of same Krull dimension. Its ice quiver is certain *hive quiver* Δ_n . The initial cluster, and the coefficients can be explicit described in terms of Schofield's semi-invariants [8].

When $n \leq 6$, we prove that the semi-invariant ring $SI_{\beta}(T_n)$ is exactly the cluster algebra $C(\Delta_n)$. We conjecture this is true for all n. An interesting consequence of the cluster structure is that these semi-invariant rings are generated by invariants of *extremal weights*, and thus we can explicitly describe minimal generators and even relations. We conjecture that being extremally generated holds for all n.

To show $C(\Delta_n) \subseteq SI_{\beta}(T_n)$, we use an idea similar to [5]. The hard part is to show the other containment, because we do not even know a finite set of generators of $SI_{\beta}(T_n)$. For this, we consider Knuston-Tao's hive polytopes, and show that each is (volume-preserving) isomorphic to certain polytope $G(\sigma)$ inside the (closure of) convex cone generated by g-vectors [1]. As a consequence, the lattice points of $G(\sigma)$ counts the dimension of semi-invariants of weight σ . When the cluster algebras are of finite type (n < 6), the cone is polyhedral, but in general it is not.

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Love Forsberg (Uppsala University)

Effective dimensions of truncated path semigroups

Many of the most interesting algebras are (essentially) linear spans of semigroups. Given a semigroup S and a field K we define the *effective dimension eff.dim*_KS as the least n such that an injective semigroup homomorphism $S \to \operatorname{Mat}_{n \times n}(\mathbb{K})$ exists. The effective dimension is in general hard to calculate, but is finite for finite semigroups. We show that the effective dimension of truncated path semigroups depends nontrivially on the structure of the underlying quiver.

Volodymyr Gavran (NAS of Ukraine)

Cohen-Macaulay modules over non-commutative surface singularities

In [1] and [2] M. Artin completely classified two-dimensional maximal orders of finite Cohen-Macaulay type. However the problem of classification of Cohen-Macaulay *tame* 2-dimensional orders is quite hard and still remains open. In [4] we give the possible approach to deal with this problem. Namely, we generalize the results of C. Kahn [5] about a correspondence between Cohen-Macaulay modules and vector bundles to maximal orders. The crucial ingredient for this correspondence is the recent result of Chan-Ingalls [3] about resolution of noncommutative singularities. As an application, we give examples of maximal orders for which the classification problem of CM modules reduces to classification of vector bundles over an elliptic curve. Thus they are not Cohen-Macaulay finite, but are Cohen-Macaulay tame.

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Shengfei Geng (Sichuan University)

An embedding from the twist Ringel-Hall algebra to the Bridgeland's Ringel-Hall algebra

Let $\mathcal{A} = \text{modA}$ be the category of finitely generated right A-modules, where A is a finite-dimensional algebra with finite global dimension. Recently, Bridgeland in [Br] introduces a localized Hall algebra $\mathcal{DH}(\mathcal{A})$ and a reduced localized Hall algebra $\mathcal{DH}_{\text{red}}(\mathcal{A})$ to recover the whole quantized enveloping algebra. When A is a hereditary algebra, Bridgeland proves that there is an embedding from the twist Ringel-Hall algebra $\mathcal{H}_{\text{tw}}(\mathcal{A})$ to the Bridgeland's Hall algebra $\mathcal{DH}(\mathcal{A})$. During this report, I will talk the following two problems:

(1) When A has global dimension ≤ 2 , we show that there is an embedding from the twisted Ringel-Hall algebra to the Brigeland's Ringel-Hall algebra. In particular, this result is true for tilted algebras and canonical algebras. This is joint work with Liangang Peng, See [GP].

(2) By proper Gorenstein projective resolution, we define twisted Ringel Hall algebra and the Brigeland's Hall algebra over d Gorenstein algebras A, where the definitions are concise with the definitions if A has finite global dimension. When Ais 1 Gorenstein algebra, we also prove that there is an embedding from the twisted Ringel Hall algebra to the Brigeland's Hall algebra. Since cluster tilted algebras have Gorenstein dimension 1, the embedding is still true over cluster tilted algebras. This is joint work with Xiaojuan Zhao.

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Christof Geiss (Instituto de Matematicas, UNAM) Geometric construction of non-simply laced simple Lie algebras

This is a report on joint work in progress with B. Lecerc and Jan Schröer.

A. Schofield showed in a 1994 manuscript how to construct the enveloping algebra $U(\mathfrak{n}_+)$ of the positive part of a Kac-Moody algebra with symmetric Cartan matrix in terms of constructible functions on the representation spaces of a quiver of the corresponding type.

We extend this result to symmetrizable Cartan matrices of positive type (corresponding thus to the Dynkin types $B_{n\geq 2}$, $C_{n\geq 3}$, F_4 and G_2). A central role in this approach is played by a, apparently new, class of 1-Gorenstein algebras $H(C, \Omega)$ defined for any symmetrizable Cartan matrix C and any orientation Ω . These algebras can be defined over any algebraically closed field and model sufficiently well the representation theory of the corresponding species. In particular, in the Dynkin case we find for each positive root a unique rigid indecomposable representation.

If time permits, we discuss briefly a "preprojective" algebra $\Pi(C)$ for $H(C, \Omega)$ which should allow to extend Lusztig's construction of a semicanonical basis of $U(\mathfrak{n}_+)$ to this setting.

Lorna Gregory (The University of Manchester) Interpretation functors, wild algebras and undecidability

An interpretation functor $I : Mod - R \to Mod - S$ is an additive functor which commutes with products and direct limits.

We show that if \mathcal{A} is a finitely controlled wild k-algebra then there is an interpretation functor $I : \operatorname{Mod} - \mathcal{A} \to \operatorname{Mod} - k\mathbb{K}_3$ whose image is the whole of $\operatorname{Mod} - k\mathbb{K}_3$. From this we deduce that the theory of $\operatorname{Mod} - \mathcal{A}$ interprets the theory of $\operatorname{Mod} - k\mathbb{K}_3$ and thus when k is countable, that the theory of $\operatorname{Mod} - \mathcal{A}$ is undecidable. For general wild algebras \mathcal{A} , we show that for any finite dimensional algebra \mathcal{B} , there is an embedding of the lattice of pp-1-formulae of \mathcal{B} into the lattice of pp-n-formulae of \mathcal{A} for some $n \in \mathbb{N}$.

This is joint work with Mike Prest.

Jin Yun Guo (Hunan normal university) On *n*-translation algebras

In this paper, we introduce *n*-translation quivers and *n*-translation algebras, aimed at generalizing certain classical results to higher representation theory. The classical $\mathbb{Z}Q$ construction of the translation quiver is generalized to construct an (n + 1)translation quiver from an *n*-translation quiver, using trivial extension and smash product. We prove that the quadratic dual of *n*-translation algebras have (n - 1)almost splitting sequences in the category of its projective modules. We also present a non-Koszul 1-translation algebra whose trivial extension is 2-translation algebra, thus also give a class of examples of (3, m - 1)-Koszul algebras (and also a class of (m - 1, 3)-Koszul algebras) for all $m \geq 2$.

Yang Han (Chinese Academy of Sciences) Brauer-Thrall type theorems for derived category

This is a joint work with Chao Zhang [Ref. arXiv:1310.2777].

The numerical invariants (global) cohomological length, (global) cohomological width, and (global) cohomological range of a complex (an algebra) are introduced. Cohomological range leads to the concepts of derived bounded algebra and strongly derived unbounded algebra naturally. The first and second Brauer-Thrall type theorems for the bounded derived category of a finite-dimensional algebra over an algebraically closed field are obtained. The first Brauer-Thrall type theorem says that derived bounded algebras are just derived finite algebras. The second Brauer-Thrall type theorem says that an algebra is either derived discrete or strongly derived unbounded, but not both. Moreover, piecewise hereditary algebras and derived discrete

algebras are characterized as the algebras of finite global cohomological width and finite global cohomological length respectively.

Reiner Hermann (Norwegian University of Science and Technology) Triangular algebras, recollements and the Gerstenhaber bracket in Hochschild cohomology

As first observed by Happel in the special case of one-point-extensions, and subsequently by Green-Solberg (and others) in the general situation, there is a long exact sequence which connects the Hochschild cohomology of a triangular algebra with the Hochschild cohomologies of its component algebras. It is known, that the morphisms in this sequence are multiplicative, and we will show that they are actually morphisms of strict Gerstenhaber algebras, i.e., that they not only preserve the multiplicative structure, but the Lie bracket and the squaring map as well. In fact, we will even go a little step further, in that we show that the strict Gerstenhaber structure is preserved whenever we deal with a recollement situation between module categories.

Our proof substantially relies on a generalisation of Schwede's exact sequence interpretation of the Lie bracket in Hochschild cohomology. Time permitting, a brief overview thereof will be given.

Martin Herschend (Uppsala University) 2-representation finite algebras from one dimensional hypersurfaces

In ongoing joint work with Osamu Iyama, Ryo Takahashi and Kota Yamaura we study tilting theory for one dimensional hypersurfaces. More specifically, let S be the polynomial algebra in two variables with standard grading and $f_1, \dots, f_n \in S$ be linear forms generating distinct ideals in S. We show that the stable category of graded Cohen-Macaulay modules over $R := S/(f_1 \dots f_n)$, denoted $\underline{CM}^{\mathbb{Z}}R$, has a tilting object U, which is analogous to a cluster tilting object found in [BIKR]. Moreover, $\underline{CM}^{\mathbb{Z}}R$ is triangle equivalent to $K^b(\text{proj}\Lambda)$, where $\Lambda := \text{End}_{\underline{CM}^{\mathbb{Z}}R}(U)$.

It turns out that Λ is 2-representation finite in the sense of [IO]. Thus we obtain a previously unknown infinite family of 2-representation finite algebras depending on parameters corresponding to the choice of linear forms. By [HI] the 3-preprojective algebra Π of Λ is isomorphic to the Jacobian algebra of a selfinjective quiver with potential. In our setting Π is isomorphic to $\operatorname{End}_{\underline{CM}^{\mathbb{Z}/2\mathbb{Z}}R}(U)$. I will present descriptions of $\operatorname{End}_{R}^{\mathbb{Z}}(U)$ and $\operatorname{End}_{R}^{\mathbb{Z}/2\mathbb{Z}}(U)$ using a graded ice quiver with potential. By removing the frozen vertices we obtain descriptions of the corresponding stable endomorphism algebras, which are isomorphic to Λ and Π respectively.

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Jun Hu (Beijing Institute of Technology) *Quiver Schur algebras*

In this talk, I will introduce some latest progress in the graded representation theory of cyclotomic quiver Schur algebras which are based on some joint work with Andrew Mathas.

Ayako Itaba (Tokyo University of Science)

On Hochschild cohomology of a self-injective special biserial algebra obtained by a circular quiver with double arrows

This talk is based on [I]. In this talk, for an integer $s \geq 3$, we calculate the dimensions of the Hochschild cohomology groups of a self-injective special biserial algebra Λ_s obtained by a circular quiver with s vertices and 2s arrows. Moreover, we give a presentation of the Hochschild cohomology ring modulo nilpotence of Λ_s by generators and relations. This result shows that the Hochschild cohomology ring modulo nilpotence of Λ_s is finitely generated as an algebra.

Note that the algebra Λ_1 is the exterior algebra in two variables $K[x, y]/\langle x^2, xy + yx, y^2 \rangle$, and Xu and Han have studied the Hochschild cohomology of the exterior algebras in arbitrary variables in [XH]. Also, in [ST] and [F], the Hochschild cohomology groups and rings for classes of some self-injective special biserial algebras have been studied. We notice that these classes contain algebras isomorphic to our algebras Λ_2 and Λ_4 .

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Tomohiro Itagaki (Tokyo University of Science)

The dimension formula of the cyclic homology of truncated quiver algebras over a field of positive characteristic

This talk is based on joint work with Katsunori Sanada. In this talk, we show the dimension formula of the cyclic homology of truncated quiver algebras over an arbitrary field.

In [3], for a truncated quiver algebra A over a commutative ring, Sköldberg gives a left A^e -projective resolution of A and computes the Hochschild homology $HH_n(A)$. By means of this result and a theorem in Loday's book (1992), Taillefer [4] gives a dimension formula of the cyclic homology of truncated quiver algebras over a field of characteristic zero.

We compute the dimension formula of the cyclic homology of truncated quiver algebras over an arbitrary field by means of chain maps in [1] and a spectral sequence. Our result generalizes the result of Taillefer into the case that the ground field is a field of any characteristic.

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Alexander Ivanov (Saint Petersburg State University)

BV-algebra structure on the Hochschild cohomology ring of the quaternion group of order eight in characteristic two

Let k be an algebraically closed field of characteristic two and let Q_8 be the quaternion group of order 8. We determine the Gerstenhaber Lie algebra structure and the Batalin-Vilkovisky structure over the Hochschild cohomology ring of the group algebra kQ_8 . This is joint work with Sergei Ivanov, Yuriy Volkov, and Guodong Zhou.

Osamu Iyama (Nagoya University)

Cohen-Macaulay representations of Geigle-Lenzing complete intersections

Weighted projective lines introduced by Geigle-Lenzing in 1987 is one of the fundamental objects in representation theory. They are derived equivalent to canonical algebras introduced by Ringel in 1984. As a higher dimensional g eneralization of weighted projective lines, we introduce a class of commutative rings R graded by abelian groups L, which we call Geigle-Lenzing complete intersections. We will study L-graded Cohen-Macaulay R-modules, and show that there always exists a tilting object in the stable category. As an application we study when (R, L) is d-representation finite in the sense of higher dimensional Auslander-Reiten theory. This is a joint work with Herschend, Minamoto and Oppermann.

Gustavo Jasso (Nagoya University) *n*-Abelian categories and *n*-exact categories

In this talk we will introduce *n*-abelian and *n*-exact categories, where *n* is a positive integer. These categories are higher analogs of abelian respectively exact categories from the viewpoint of the length of the sequences involved. Our aim is to provide a categorical framework for the investigation of the intrinsic homological properties of *n*-cluster-tilting subcategories of abelian categories and exact categories. We will also introduce Frobenius *n*-exact categories and explain their connection with Geiß-Keller-Oppermann's (n + 2)-angulated categories [1].

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Martin Kalck (Bielefeld University) Spherical subcategories and a new invariant for triangulated categories

This is joint work with A. Hochenegger and D. Ploog. Motivated by examples arising in algebraic geometry, we study objects of k-linear triangulated categories with two-dimensional graded endomorphism algebra. Given such an object, we show that there is a *unique maximal* triangulated subcategory, in which the object is spherical, i.e. a Calabi-Yau object. In many examples, both from representation theory and geometry these *spherical subcategories* admit explicit descriptions. Furthermore, the collection of all spherical subcategories ordered by inclusion yields a new invariant for triangulated categories. We derive coarser invariants like height, width and cardinality of this poset and provide some evidence that these invariants reflect important properties of triangulated categories.

Noritsugu Kameyama (Shinshu University)

Group-graded and group-bigraded rings

This talk is besed on joint work with Mitsuo Hoshino and Hirotaka Koga [2]. Let I be a non-trivial finite multiplicative group with the unit element e and $A = \bigoplus_{x \in I} A_x$ an I-graded ring. We construct a Frobenius extension Λ of A and study when the ring extension A of A_e can be a Frobenius extension. Also, formulating the ring structure of Λ , we introduce the notion of I-bigraded rings and show that every I-bigraded ring is isomorphic to the I-bigraded ring Λ constructed above.

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Ryo Kanda (Nagoya University)

Specialization orders on atom spectra of Grothendieck categories

The aim of this talk is to provide systematic methods to construct Grothendieck categories with remarkable structures and to establish a theory of the specialization orders on the spectra of Grothendieck categories.

In commutative ring theory, Hochster characterized topological spaces appearing as the prime spectra of commutative rings ([Hoc69, Theorem 6 and Proposition 10]). Speed [Spe72] pointed out that Hochsters result gives the following characterization of partially ordered sets appearing as the prime spectra of commutative rings.

Theorem 1 (Hochster [Hoc69, Proposition 10] and Speed [Spe72, Corollary 1]). Let P be a partially ordered set. Then P is isomorphic to the prime spectrum of some commutative ring with the inclusion relation if and only if P is an inverse limit of finite partially ordered sets in the category of partially ordered sets.

In [Kan12a] and [Kan12b], we investigated Grothendieck categories by using the associated topological spaces called the atom spectra of them. For a Grothendieck category \mathcal{A} , the atom spectrum ASpec \mathcal{A} has a partial order. For a commutative ring R, the partially ordered set ASpec(ModR) is isomorphic to the prime spectrum SpecR with the inclusion relation. Hence we can consider that the atom spectrum of a Grothendieck category is a (noncommutative) generalization of the prime spectrum of a commutative ring, and it is natural to ask which partially ordered sets appear as the atom spectra of Grothendieck categories.

In order to answer this question, we introduce a construction of Grothendieck categories by using colored quivers. A sextuple (Q_0, Q_1, C, s, t, u) is called a *colored quiver* if (Q_0, Q_1, s, t) is a quiver (not necessarily finite), C is a set (of colors), and

 $u: Q_1 \to C$ is a map. From a colored quiver which satisfies some condition of local finiteness, we construct a Grothendieck category associated to the colored quiver. By using this construction, we can show the following result, which is a complete answer to the above question.

Theorem 2 ([Kan13]). For any partially ordered set P, there exists a Grothendieck category \mathcal{A} such that the atom spectrum $ASpec\mathcal{A}$ is isomorphic to P as a partially ordered set.

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Yuta Kimura (Nagoya University)

Tilting objects in stable categories of factor algebras of preprojective algebras

Let Q be a finite acyclic quiver. For an element w in the Coxeter group of Q, Buan-Iyama-Reiten-Scott introduced the factor algebra Π_w of the preprojective algebra of Q. They showed that the stable category of submodules of free Π_w -modules has a cluster tilting object.

In this talk, we regard Π_w as a graded algebra and show that the stable category of graded submodules of free Π_w -modules has a tilting object if w is a Coxeter sortable element.

Hirotaka Koga (University of Tsukuba)

Derived equivalences and Gorenstein dimension

A ring A is said to be left (resp., right) coherent if every finitely generated left (resp., right) ideal of it is finitely presented (see [3]). Let A, B be derived equivalent left and right coherent rings (see [5]). In [4] Kato showed that a standard derived equivalence induces an equivalence between the triangulated categories consisting of complexes of finite Gorenstein dimension and that a derived equivalence induces an equivalence between the projectively stable categories of modules of Gorenstein dimension zero (see [1] and [2]) if either inj dim $A < \infty$ or inj dim $A^{op} < \infty$. In this talk, we provide

alternative proofs of these results from another point of view. Also, we do not assume the existence of standard derived equivalence or finiteness of selfinjective dimension.

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Masahide Konishi (Nagoya University)

How to basicalize KLR algebras

KLR algebras are introduced to categorify the negative part of the associated quantum group. From another point of view, they are a class of infinite dimensional algebra, defined by two data: a quiver and a weight on its vertices. It is not so pathological, therefore we know we can obtain a basic algebra Morita equivalent to a KLR algebra as a quiver with relations in principle. In this talk, I will explain an explicit algorithm for that in some special cases.

Justyna Kosakowska (Nicolaus Copernicus University)

The boundary of the irreducible components for invariant subspace varieties

Given partitions α, β, γ , the short exact sequences

$$0 \longrightarrow N_{\alpha} \longrightarrow N_{\beta} \longrightarrow N_{\gamma} \longrightarrow 0$$

of nilpotent linear operators of Jordan types α, β, γ , respectively, define a constructible subset $\mathbb{V}^{\beta}_{\alpha,\gamma}$ of an affine variety.

Geometrically, the varieties $\mathbb{V}_{\alpha,\gamma}^{\beta}$ are of particular interest as they occur naturally and since they typically consist of several irreducible components. In fact, each Littlewood-Richardson tableaux Γ of shape (α, β, γ) contributes one irreducible component $\overline{\mathbb{V}}_{\Gamma}$.

We consider the partial order $\Gamma \leq_{\text{bound}}^* \tilde{\Gamma}$ on LR-tableaux which is the transitive closure of the relation given by $\mathbb{V}_{\tilde{\Gamma}} \cap \overline{\mathbb{V}}_{\Gamma} \neq \emptyset$. In this paper we compare the boundary-relation with partial orders given by algebraic, combinatorial and geometric conditions. In the case where the parts of α are at most two, all those partial orders are equivalent. We prove that those partial orders are also equivalent in the case where $\beta \setminus \gamma$ is a horizontal and vertical strip. Moreover, we discuss how the orders differ in general.

This is a talk about a joint project with Markus Schmidmeier from Florida Atlantic University.

Henning Krause (Bielefeld University)

Non-crossing partitions and hereditary artin algebras

Any classification of reflection groups or root systems raises the obvious question of how different types are related. We present an answer based on non-crossing partitions. This involves a category of certain bilinear lattices, which are essentially determined by a symmetrisable generalised Cartan matrix together with a particular choice of a Coxeter element. Generic examples arise from Grothendieck groups of hereditary artin algebras. This is joint work with Andrew Hubery.

Julian Külshammer (University of Stuttgart)

In the box seat: Quasi-hereditary algebras

This is joint work with Steffen König, Sergiy Ovsienko. Motivated by a prototypical example of quasi-hereditary algebras - the blocks of Bernstein-Gelfand-Gelfand category \mathcal{O} - König introduced the notion of an exact Borel subalgebra B of a quasihereditary algebra A. These algebras reflect the properties of the universal enveloping algebra of a Borel subalgebra of a complex semisimple Lie algebra \mathfrak{g} . In particular a version of the PBW theorem holds and simple B-modules are induced to standard modules for A. Before proving the following main theorem, existence of exact Borel subalgebras was unknown for many famous classes of quasi-hereditary algebras like Schur algebras or algebras of global dimension two.

Theorem. Up to Morita equivalence, every quasi-hereditary algebra has an exact Borel subalgebra. More precisely, an algebra A is quasi-hereditary if and only if it is Morita equivalent to an algebra which can be constructed as the right Burt-Butler algebra of a directed box.

Here, boxes (bimodules over categories with coalgebra structures) are certain generalisations of algebras which have been studied in the context of Drozd's tame and wild theorem. The analogue of an exact Borel subalgebra in the language of boxes is what we call a directed box.

The language of boxes enables us to recover much of the nice theory of quasihereditary algebras like the existence of almost split sequences, the characteristic tilting module, and Ringel duality. Moreover, the induction functor from the exact Borel subalgebras coming with the boxes, satisfies additional properties: It preserves higher Ext-groups and almost split sequences. **Rosanna Laking** (University of Manchester) Morphisms in $K^b(\text{proj}(\Lambda))$ for Λ a gentle algebra

This talk will present joint work with Kristin Krogh Arnesen (NTNU) and David Pauksztello (Manchester).

We identify a basis for $\operatorname{Hom}_{K^b(\operatorname{proj}(\Lambda))}(P,Q)$ where P,Q are any indecomposable objects in $K^b(\operatorname{proj}(\Lambda))$ and Λ is a gentle algebra. This description uses the language of homotopy strings and homotopy bands introduced by Bekkert and Merklen. We provide a method for calculating the dimension of $\operatorname{Hom}_{K^b(\operatorname{proj}(\Lambda))}(P,Q)$ based entirely on the combinatorics of the homotopy strings or bands associated to P and Q and provide some examples to illustrate this method.

Philipp Lampe (Bielefeld University)

The divisor class group of a cluster algebra

When we categorify a cluster algebra via representation theory, its algebraic structure plays a crucial role. In this talk, we prove that if the cluster variables in a single acyclic cluster are prime elements, then the cluster algebra is a unique factorization domain. Our main tool is its attached divisor class group. As an application, we investigate which finite type cluster algebras are unique factorization domains. We conclude by discussing further examples of divisor class groups and by stating a conjecture about torsion.

Helmut Lenzing (Universität Paderborn)

On piecewise hereditary Nakayama algebras

By $A_n(r)$ we denote the quotient of the path algebra of the equi-oriented linear quiver \mathbb{A}_n , arrows denoted by x, by all relations $x^r = 0$. Completing research by Happel and Seidel [3] we give, in a short communication, a complete description of all piecewise hereditary Nakayama algebras $A_n(r)$ and of their hereditary types. Our proof involves perpendicular calculus ([2], [4], see also [1]) and joint work with J.A. de la Peña on wild canonical algebras [5] and Fuchsian singularities [6].

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Qunhua Liu (Nanjing Normal University) Glueing t-structures and tilting objects via recollements

The concept of recollement of triangulated categories was introduced by Beilinson, Bernstein and Deligne in 1982. They also developed a technique of glueing *t*-structures via recollements for the construction of perverse sheaves. In this talk we first provide a necessary and sufficient condition of which *t*-structures can be obtained by glueing, for piecewise hereditary algebras [LV]. Under the correspondence between *t*-structures and silting objects due to Koenig and Yang, glueing *t*-structures corresponds to glueing silting objects. We then provide a necessary and sufficient condition under which the glueing of two tilting objects yields a tilting object [LVY].

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Yu Liu (Nagoya University) Hearts of twin cotorsion pairs on exact categories

The cotorsion pairs were first introduced by Salce, and it has been deeply studied in the representation theory during these years, especially in tilting theory and Cohen-Macaulay modules. Recently, the cotorsion pairs are also studied in triangulated categories [2], in particular, Nakaoka introduced the notion of hearts of cotorsion pairs and showed that the hearts are abelian categories [4]. This is a generalization of the hearts of t-structure in triangulated categories [1] and the quotient of triangulated categories by cluster tilting subcategories [3]. Moreover, he generalized these results to a more general setting called twin cotorsion pair [5].

The aim of this paper is to give similar results for cotorsion pairs on Quillen's exact categories, which plays an important role in representation theory. We consider a *cotorsion pair* in an exact category, which is a pair $(\mathcal{U}, \mathcal{V})$ of subcategories of an exact category \mathcal{B} satisfying $\operatorname{Ext}^1_{\mathcal{B}}(\mathcal{U}, \mathcal{V}) = 0$ (*i.e.* $\operatorname{Ext}^1_{\mathcal{B}}(\mathcal{U}, \mathcal{V}) = 0, \forall \mathcal{U} \in \mathcal{U} \text{ and } \forall \mathcal{V} \in \mathcal{V}$) and any $B \in \mathcal{B}$ admits two short exact sequences $V_B \rightarrow U_B \rightarrow B$ and $B \rightarrow V^B \rightarrow U^B$ where $V_B, V^B \in \mathcal{V}$ and $U_B, U^B \in \mathcal{U}$. Let

$$\mathcal{B}^+ := \{ B \in \mathcal{B} \mid \mathcal{U}_B \in \mathcal{V} \}, \mathcal{B}^- := \{ B \in \mathcal{B} \mid \mathcal{V}^B \in \mathcal{U} \}.$$

We define the *heart* of $(\mathcal{U}, \mathcal{V})$ as the quotient category

$$\underline{\mathcal{H}} := (\mathcal{B}^+ \cap \mathcal{B}^-)/(\mathcal{U} \cap \mathcal{V}).$$

An important class of exact categories is given by Frobenius categories, which gives most of important triangulated categories appearing in representation theory. Now we state our first main result, which is an analogue of [4, Theorem 6.4].

Theorem 1. Let $(\mathcal{U}, \mathcal{V})$ be a cotorsion pair on an exact category \mathcal{B} with enough projectives and injectives. Then $\underline{\mathcal{H}}$ is abelian.

Moreover, following Nakaoka, we consider pairs of cotorsion pairs (S, \mathcal{T}) and $(\mathcal{U}, \mathcal{V})$ in \mathcal{B} such that $S \subseteq \mathcal{U}$, we also call such a pair a *twin cotorsion pair*. The notion of hearts is generalized to such pairs, and our second main result is the following, which is an analogue of [5, Theorem 5.4].

Theorem 2. Let (S, T), (U, V) be a twin cotorsion pair on \mathcal{B} . Then $\underline{\mathcal{H}}$ is semiabelian.

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Xueyu Luo (Nagoya University)

0-Calabi-Yau configurations and finite Auslander-Reiten quivers of Gorenstein Orders

In the 1980s, Wiedemann developed a classification theory of Auslander-Reiten quivers of representation-finite Gorenstein orders which is similar but different from Riedtmanns work on representation-finite self-injective algebras. Following Wiedemanns classification [1], those Auslander-Reiten quivers can be interpreted in terms of a Dynkin diagram, a configuration and an automorphism group. Wiedemanns configurations are "0-Calabi-Yau" in the sense that they are stable under the Serre functor in the stable category of Cohen-Macaulay modules, while Riedtmanns configurations are "(-1)-Calabi-Yau". We introduce the notion of 2-*Brauer relations* and simplified Wiedemanns description of his configurations.

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Piotr Malicki (Nicolaus Copernicus University) Hochschild cohomologies of generalized multicoil algebras

This is report on joint work with A. Skowroński. Let A be a basic, finite dimensional k-algebra over a fixed algebraically closed field k, modA the category of finite dimensional right A-modules, and Γ_A the Auslander-Reiten quiver of A. Let \mathcal{C} be a connected component of Γ_A . Recall that \mathcal{C} is called *almost cyclic* if all but finitely many modules of \mathcal{C} lie on oriented cycles (in \mathcal{C}). Further, \mathcal{C} is called *coherent* if every projective module in \mathcal{C} is the starting module of an infinite sectional path and every injective module in \mathcal{C} is the ending module of an infinite sectional path. Moreover, a family $\mathcal{C}_A = (\mathcal{C}_i)_{i \in I}$ of components of Γ_A is called *separating* in modA if the indecomposable modules in modA split into three disjoint classes $\mathcal{P}_A, \mathcal{C}_A$ and \mathcal{Q}_A such that:

- (S1) \mathcal{C}_A is sincere and generalized standard;
- (S2) $\operatorname{Hom}_{A}(\mathcal{Q}_{A}, \mathcal{P}_{A}) = 0$, $\operatorname{Hom}_{A}(\mathcal{Q}_{A}, \mathcal{C}_{A}) = 0$, $\operatorname{Hom}_{A}(\mathcal{C}_{A}, \mathcal{P}_{A}) = 0$;
- (S3) any morphism from \mathcal{P}_A to \mathcal{Q}_A factors through $\mathrm{add}(\mathcal{C}_A)$.

The aim of the talk is to present the Hochschild cohomologies of all finite dimensional generalized multicoil algebras over an algebraically closed field, which are the algebras for which the Auslander-Reiten quiver admits a separating family of almost cyclic coherent components. As an application we receive a description of the analytically rigid generalized multicoil algebras.

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František Marko (Pennsylvania State University Hazleton) The center of Dist(GL(m|n)) in positive characteristic¹

This is joint work with Alexandr N. Zubkov. We investigate central elements in distribution algebras Dist(G) of general linear supergroups G = GL(m|n) over a ground field K of positive characteristic. As an application, we compute explicitly

the center of Dist(GL(1|1)) and its image under Harish-Chandra homomorphism. We also compare the usual blocks, the Harish-Chandra blocks [1] and Kujawa blocks [3] for GL(1|1). The main motivation of this work was the paper [2] of Haboush who related the adjoint invariants to central elements of the distribution algebra of a semisimple simply connected algebraic group over the field of positive characteristic. Our main goal was to extend his results and method to the case of supergroups.

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Hagen Meltzer (Institute of Mathematics Szczecin) Matrix factorizations for domestic triangle singularities

This is a report on joint work with Dawid Kędzierski and Helmut Lenzing. We study \mathbb{L} graded matrix factorizations for singularities given by the equation $f = x_1^a + x_2^b + x_3^c$, where the sequence (a, b, c) determines a weighted projective line of domestic type and \mathbb{L} is the abelian group with generators $\vec{x_1}$, $\vec{x_2}$, $\vec{x_3}$ and relations $a\vec{x_1} = b\vec{x_2} = c\vec{x_3}$. This is related to the study of \mathbb{L} graded Cohen-Macaulay modules over the algebra $S = k[x_1x_2, x_3]/(f)$ and to the singularity category of the \mathbb{L} graded algebra S in the sense of Buchweitz and Orlov. By recent joint work with Kussin and Lenzing we have also interesting connections to stable categories of vector bundles on weighted projective lines and this fact is widely used in the present work.

Such matrix factorizations were also studied in the \mathbb{Z} graded case and by different methods by Kajiura, Saito and Takahashi. However, in contrast to them we work over an arbitrary closed field of arbitrary characteristic.

Hiroyuki Minamoto (Osaka Prefecture University) Derived bi-commutator ring and DG-completion

We discuss a derived version of bi-commutator ring (also known as double centralizer ring or bi-endomorphism ring), which is a basic notion in ring theory. Let R be a ring, J an R-module and $E := \operatorname{End}_R(J)$ the endomorphism ring. Then J has a canonical E-module structure and the endomorphism ring of J as an E-module is called the bi-commutator ring and denoted by $\operatorname{Bic}_R(J) := \operatorname{End}_E(J)$, which comes equipped with a canonical ring homomorphism $\gamma_{R,J} : R \to \operatorname{Bic}_R(J)$. It is well-known that the first commutator ring E and the tricommutator ring $\operatorname{Tric}_R(J) := \operatorname{Bic}_E(J)$ are isomorphic via the canonical morphism $\gamma_{E,J}$. Hence, for n > 1, the n-th commutator ring and n + 2-th commutator ring are canonically isomorphic. Because of this property, we may regard taking a bi-commutator ring as kind of a "closure operation".

Recently, in the study of derived categories, the concern with the derived bicommutator ring has been growing. (See, e.g., [1, 2, 3, 4, 5, 6, 7]). Therefore it is necessary to understand basic properties of the derived bi-commutator rings. In this talk, we discuss a derived version of the above property of bi-commutator rings. Namely, is the canonical morphism $\gamma : \mathbb{E}nd_R(J) \to \operatorname{Tric}_R(J)$ a quasi-isomorphism? We show by an example that the answer is negative. As a result for a positive direction, we prove that proxy smallness (introduced by Dwyer-Greenlees-Iyengar) of the center DG-module of a derived commutator ring ensures that the property holds.

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Yuya Mizuno (Nagoya university) Preprojective algebras and τ -tilting theory

Preprojective algebras were invented by Gelfand-Ponomarev in the late 1970s. Since then, they have not only been one of the important objects in the representation theory of algebras, but also in many branches of mathematics such as algebraic geometry and quantum groups. From the viewpoint of the representation theory of algebras, it is very natural to investigate the categorical structures.

Recently, Buan-Iyama-Reiten-Scott studied preprojective algebras of non-Dynkin quivers by using certain ideals. It turned out that these ideals are tilting modules and this fact gives various nice results. In this talk, we explain that τ -tilting modules, which is a generalization of tilting modules, play a central role in the case of Dynkin quivers. We show that support τ -tilting modules can be regarded as a categorical realization of the Weyl group and they provide explicit connections between various important objects in several categories.

Agustin Moreno Cañadas (National University of Colombia) Algorithms of differentiation of posets to enumerate higher dimensional partitions

This is joint work with Pedro Fernando Fernandéz Espinosa.

Recently, P. Fahr and C. M. Ringel used the preprojective component of the 3-Kronecker quiver and its cover in order to obtain a partition formula for the evenindex Fibonacci numbers [2, 3].

In this talk, we describe how it is possible to use relations between the number of indecomposable representations of a given poset \mathcal{P} and its corresponding derived poset via the algorithm of differentiation with respect to a maximal point introduced by Nazarova and Roiter to enumerate higher dimensional partitions whose parts are either polygonal numbers or generalized Fibonacci numbers [1].

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Intan Muchtadi-Alamsyah (Institut Teknologi Bandung) On Nakayama m-Cluster Tilted Algebras of Type A_n

This is joint work with Faisal. Nakayama algebras are algebras that are both right and left serial. For an algebraically closed field k, Nakayama algebras are given as path algebras of quivers of Dynkin type A_n or oriented cycle, modulo ideals generated by linear combinations of paths. The study of *m*-cluster tilted algebras has been a major research in recent years. These algebras occur as endomorphism algebras of certain objects in triangulated Hom-finite categories. Ringel has characterized self-injective 1-cluster tilted algebras which include Nakayama case. In this work we determine all *m*-cluster tilted algebras of type A_n which are also Nakayama algebras by using the geometric description of *m*-cluster categories introduced by Baur and Marsh.

Alfredo Nájera Chávez (Institut de Mathématiques de Jussieu) A 2-Calabi-Yau realization of universal cluster algebras of finite type

We give a 2-Calabi-Yau realization of cluster algebras with universal coefficients associated to cluster-finite quivers (see section 12 of [1]). These categories are defined as *completed* orbit categories associated to Nkajima categories of Dynkin quivers. Remarkably, they are Ext-finite Frobenius categories with infinite-dimensional morphism spaces. We use this realization to describe the quiver of any seed of the corresponding universal cluster algebra and relate the universal coefficients with the c- and g-vectors.

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Yann Palu (Université UPJV Amiens)

From triangulated categories to module categories via homotopical algebra

Motivated by the theory of cluster algebras, Buan-Marsh-Reiten proved that some quotients of cluster categories are module categories. More generally, some subquotients (associated with rigid objects) of some Hom-finite triangulated categories have been shown to be module categories. In their paper entitled "From triangulated categories to module categories via localisation", Aslak Buan and Robert Marsh proved that these module categories can also be recovered as certain localisations of the triangulated categories under consideration. Our aim in this talk is to give a homotopical algebra point of view on their result, based on work in progress.

Manoj Kumar Patel (National Institute of Technology, Nagaland) Generalizations of semi-injective modules and their endomorphism rings

In this paper we have studied the properties of pseudo semi-injective modules and proved the various results related with co-Hopfian, weakly co-Hopfian and directly finite modules.

David Pauksztello (University of Manchester) The contractibility of the stability manifold of a discrete derived category

The algebras with discrete derived category are a class of algebras whose derived categories are intermediate in complexity between those of finite-type hereditary algebras and tame-type hereditary algebras. However, these algebras can attain arbitrarily large global dimension. This makes such categories an ideal natural laboratory to understand aspects of derived representation theory. In this talk, we concentrate on the interplay between combinatorics and geometry. The geometrical invariant we are interested in is the Bridgeland space of stability conditions, which is a complex manifold encoding the information of the t-structures in a triangulated category. This notion arises from theoretical physics and algebraic geometry. Thus far, few examples are known, and it is an important problem to determine stability manifolds in concrete examples.

We introduce a new combinatorial invariant: the poset of silting pairs, which under suitable finiteness assumptions induces a regular CW complex. In the case of discrete derived categories we show that this CW complex is contractible. Moreover, for discrete derived categories, the silting pairs CW complex contains the same information as the stability manifold. Hence, we can also deduce the contractibility of the stability manifold for the family of discrete derived categories.

This is a report on joint work with Nathan Broomhead (Hannover) and David Ploog (Essen).

Matthew Pressland (University of Bath)

Labelled seeds, homogeneous spaces and cluster automorphisms

This talk will explain how to view the set of labelled seeds of a cluster algebra as a homogeneous space for the action of a group of mutations and permutations. We describe a particular class of equivalence relations on homogeneous spaces, with the property that their equivalence classes are given by the orbits of a subgroup of the automorphism group of the space. In the labelled seeds setting, one subgroup arising in this way can be identified with the group of cluster automorphisms (in the sense of Assem-Schiffler-Shramchenko) and another with the group of direct cluster automorphisms. This is joint work with Alastair King (see *Labelled Seeds and Global Mutations*, arXiv:1309.6579 [math.RT]).

Mike Prest (University of Manchester) *Ringel's conjecture for domestic string algebras*

This is joint work with Gena Puninski.

Butler and Ringel [2] showed that the indecomposable finite-dimensional modules over string algebras fall into two classes: string modules and band modules. Over finite-dimensional algebras the pure-injective modules are the direct summands of direct products of finite-dimensional modules. Supported on each band there are infinite-dimensional indecomposable pure-injective modules: Prüfer and adic modules and a generic (see [5], [9], [3]). What other indecomposable pure-injectives are there?

Ringel [8] constructed, from each eventually periodic infinite string, an indecomposable pure-injective module and conjectured (see [10]) that these modules, together with the finite-dimensional modules and the infinite-dimensional band modules, form the complete list of indecomposable pure-injective modules over domestic string algebras (= those string algebras with only finitely many bands).

There have been persistent attempts to settle this conjecture and associated questions (e.g. [1] and see [11]), most recently in the thesis of Harland [4] and by the first author who completed the case of 1-domestic string algebras [6].

We have shown [7] that Ringel's conjecture is correct. We are able to reduce it, using some of Harland's results, to the already known 1-domestic case.

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Marju Purin (St. Olaf College)

The generalized Auslander-Reiten condition for n-symmetric algebras

A ring R is said to satisfy the Generalized Auslander-Reiten Condition if for each R-module M with $\operatorname{Ext}_{R}^{i}(M, M \oplus R) = 0$ for all i > n the projective dimesion of M is at most n. We prove that this condition holds for all n-symmetric algebras of quasitilted type (a broad class of self-injective algebras where every module is ν -periodic). Here ν denotes the Nakayama automorphism. This is joint work with M. Karpicz.

Udhayakumar Ramalingam (Periyar University)

Gorenstein n-flat modules and their covers

In this talk, we introduce the notion of Gorenstein *n*-flat modules and Gorenstein n-absolutely pure modules. First, we prove that the direct limit of Gorenstein n-flat modules over a right n-coherent ring is again a Gorenstein n-flat module. Also we prove that over a right n-coherent ring, any pure submodule of a Gorenstein n-flat module is a Gorenstein n-flat module. Finally, the class of all Gorenstein n-flat left modules over a right n-coherent ring have Gorenstein n-flat covers. Some applications are also given.

Rebecca Reischuk (Bielefeld University)

The monoidal structure of modules over Schur algebras

The category of strict polynomial functors inherits an internal tensor product from the category of divided powers. This yields a monoidal structure for the category of modules over the Schur algebra $S_k(n, d)$. To investigate this monoidal structure, we consider the category of modules over the symmetric group algebra $k\mathfrak{S}_d$ which admits a tensor product coming from its Hopf algebra structure. Based on work by Schur, there exists a functor \mathcal{F} going from the category of modules over the Schur algebra to those over the symmetric group. We show that the tensor product coming from strict polynomial functors is mapped under \mathcal{F} to the one in the category of modules over $k\mathfrak{S}_d$.

This is joint work with Cosima Aquilino.

Claus Michael Ringel (Bielefeld University)

The Auslander varieties

Let k be an algebraically closed field and A a finite-dimensional k-algebra. Given an A-module M, the set $G_e(M)$ of all submodules of M with dimension vector e is called a quiver Grassmannian. Let us stress that quiver Grassmannians are projective varieties (and that every projective variety occurs in this way).

If C, Y are A-modules, then we consider $\operatorname{Hom}(C, Y)$ as a B-module, where B is the opposite of the endomorphism ring of C, and the Auslander varieties for A are the quiver Grassmannians of the form $G_e\operatorname{Hom}(C, Y)$. The Auslander varieties should be considered as part of a basic description of the category modA, this is an essential feature of Auslander's theory of "morphisms determined by modules". The first aim of the lecture will be to point out this setting.

Secondly, we will show that if A is a controlled wild algebra, then any projective variety can be realized as an Auslander variety for A.

Shiquan Ruan (Xiamen University)

Tilting bundles and the missing part on a weighted projective line of type (2, 2, n)

We investigate the tilting bundles, i.e. tilting sheaves that are vector bundles, in the category of coherent sheaves on a weighted projective line of type (2, 2, n). We classify all the tilting bundles. Consequently, we actually obtain the classification of all concealed algebras of type $\widetilde{\mathbb{D}}_n$, which was listed by Happel-Vossiek. For each tilting bundle T with endomorphism algebra Λ , we prove that the corresponding missing part, from the category of coherent sheaves to the category of finitely generated right Λ -modules, carries the structure of an abelian category. We also give some counterexamples to show that the missing part is not, in general, abelian if the weight type is different from (2, 2, n), or if the tilting sheaf contains a finite length sheaf as a direct summand.

Shokrollah Salarian (University of Isfahan and IPM) On the Auslander-Reiten conjecture for algebras

This is joint work with A. Bahleken and A. Mahin Fallah. A recent result of Araya asserts that if the Auslander-Reiten conjecture holds in codimension one for a commutative Gorenstein ring R, then it holds for R. In this talk we extend this result to R-algebras Λ which are finitely generated free over R, whenever R is a commutative Gorenstein ring. This, in particular, implies that any finitely generated self-orthogonal Gorenstein projective Λ -module is projective, provided Λ is an isolated singularity and dim $R \geq 2$. Also, some examples of bound quiver algebras satisfying the Auslander-Reiten conjecture are presented.

Manuel Saorín (Universidad de Murcia - Spain) An axiomatic approach to degenerations in triangulated categories

The classical theory of degenerations of finitely generated modules over finite dimensional algebras (see [1]) was later approached by Yoshino from a different point of view which allowed him to extend it to modules over arbitrary (associative unital) algebras (see [2], [3]). In particular, he proved that the algebraic characterization of degenerations due to Zwara [4], after the earlier work of Riedtmann [1], was also valid in this more general context.

In this talk, we will present a recent work on which we give an axiomatic definition of degeneration of objects of any triangulated category, modelled upon Yoshino's approach. We then show that the so defined degeneration implies the triangulated analogue of the algebraic condition of Riedtmann-Zwara and the converse is also true for a wide class of triangulated categories, which includes the perfect derived category of any algebra and the bounded derived category of an Artin algebra. Our definition of degeneration, when defined in a skeletally small triangulated category, naturally defines a preorder in the set of isoclasses of its objects. At the end of the talk we will discuss conditions under which it is a partial order.

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Julia Sauter (Bielefeld University) Quiver flag varieties of finite type

We classify all pairs (Q, n) with all Q-quiver flag varieties of length n have finitely many orbits. For Dynkin quiver flag varieties we discuss when the stratum with given isomorphism type of the submodule is dense. If time permits, we explain an affine cell decomposition for uniserial situations (for example $Q = A_n$ -equioriented).

Chelliah Selvaraj (Periyar University) Stability of Gorenstein n-flat modules

In this paper, we are concerned with the stability of the class of Gorenstein *n*-flat modules. We give an answer for the following natural question in the setting of a left GF_n -closed ring R: Given an exact sequence of Gorenstein *n*-flat R-modules $\mathbf{G} = \cdots \to G_1 \to G_0 \to G^0 \to G^1 \to \cdots$ such that the complex $H \otimes_R \mathbf{G}$ is exact for each Gorenstein *n*-absolutely pure right R-module H, is the module $M := \text{Im}(G_0 \to G^0)$ a Gorenstein *n*-flat module?

Peng Shan (University of Caen) On the center of quiver Hecke algebras

In this talk, we will explain an action of the loop algebra associated with a Kac-Moody algebra on the center and cocenter of quiver Hecke algebras, and relate the latter to Weyl modules and cohomology of quiver varieties. Well also explain how to apply this result to prove the positivity of the grading on the center of quiver Hecke algebras. This is a joint work with Michela Varagnolo and Eric Vasserot.

Adam Skowyrski (Nicolaus Copernicus University)

Algebras having finitely many short paths with injective source and projective target

Let A be an artin algebra over a fixed commutative artin ring K. We denote by mod A the category of finitely generated right A-modules, and by ind A its full subcategory formed by all indecomposable modules.

We are interested in the structure of module categories modA, such that for all but finitely many isomorphism classes of modules X in indA, we have:

(IP) $\operatorname{Hom}_A(D(A), X) = 0 \text{ or } \operatorname{Hom}_A(X, A) = 0.$

There is still an open problem (formulated by Skowroński) to decide if this class of algebras consists of quasitilted algebras and generalized double tilted algebras. Note also that both quasitilted and generalized double tilted algebras satisfy the above mentioned property.

Our aim is to confirm the above conjecture for the class of cycle-finite algebras. Moreover, we show that algebras satisfying the condition (IP) for all modules X in indA form a particular subclass of the class of quasitilted algebras.

Johan Steen (NTNU, Trondheim, Norway) Strong generators in tensor triangulated categories

Let T denote an essentially small rigid tensor triangulated category. Recall that an object g of T generates strongly if there is an integer n such that every object of T can be built from g by taking suspensions, finite direct sums and summands, and at most n cones. Recall also that a subcategory $S \subseteq T$ is a *thick tensor ideal* if it is a thick subcategory and $t \otimes s \in S$ for any $t \in T$ and $s \in S$.

We show that connectedness of the Balmer spectrum, SpcT, implies that no nonzero and proper thick tensor ideals are strongly generated.

In particular it follows that every non-zero and proper thick subcategory of perfect complexes over a commutative ring does not admit a strong generator, provided the ring is connected. Another example is the finite stable homotopy category (or Spanier-Whitehead category); here the only strongly generated thick subcategory is the zero subcategory.

This is joint work with Greg Stevenson.

Xiuping Su (University of Bath)

A categorification of Grassmannian cluster algebras

Joint with A. King and B. T. Jensen. We study the category CM(A) of Cohen-Macualay modules of a twisted group ring A = G * R constructed from the coordinate ring R of the singularity $x^k = y^{n-k}$. We show that $CM(\hat{A})$ categorifies the Grassmannian cluster algebra of the same type as the singularity. The proof uses Geiss, Leclerc and Schroer's categorification of cluster algebras defined on the coordinate ring of the big cell in the corresponding Grassmannian.

Ryo Takahashi (Nagoya University)

On singularity categories of stable categories of resolving subcategories

Let R be a right noetherian ring. The *singularity category* of R is by definition the Verdier quotient

$$\mathbf{D}_{\mathrm{sg}}(R) = \mathbf{D}^{\mathrm{b}}(\mathrm{mod}R)/\mathbf{K}^{\mathrm{b}}(\mathrm{proj}(\mathrm{mod}R)),$$

where mod R stands for the category of finitely generated right R-modules, $D^{b}(modR)$ its bounded derived category, $\operatorname{proj}(modR)$ the category of finitely generated projective right R-modules, and $K^{b}(\operatorname{proj}(modR))$ its bounded homotopy category. The singularity category $D_{sg}(R)$ is a triangulated category, which has been introduced by Buchweitz [3] by the name of stable derived category and connected to the Homological Mirror Symmetry Conjecture by Orlov [9]. Many studies on singularity categories have been done so far in a lot of approaches; see [4, 5, 7, 10, 14] for instance.

In this talk we consider the singularity category of a stable resolving subcategory. Let \mathcal{A} be an abelian category with enough projective objects. Let \mathcal{X} be a resolving subcategory of \mathcal{A} , and $\underline{\mathcal{X}}$ its stable category. Then the category mod $\underline{\mathcal{X}}$ of finitely presented right $\underline{\mathcal{X}}$ -modules is an abelian category with enough projective objects [1]. Denote by $D^{b}(\text{mod}\underline{\mathcal{X}})$ the bounded derived category of $\text{mod}\underline{\mathcal{X}}$, by $\text{proj}(\text{mod}\underline{\mathcal{X}})$ the category of projective objects of $\text{mod}\underline{\mathcal{X}}$, and by $K^{b}(\text{proj}(\text{mod}\underline{\mathcal{X}}))$ the bounded homotopy category of $\text{proj}(\text{mod}\underline{\mathcal{X}})$. We take the Verdier quotient

$$D_{sg}(\underline{\mathcal{X}}) = D^{b}(mod\underline{\mathcal{X}})/K^{b}(proj(mod\underline{\mathcal{X}})),$$

and call this triangulated category the singularity category of $\underline{\mathcal{X}}$. For two resolving subcategories \mathcal{X} , \mathcal{Y} we say that $\underline{\mathcal{X}}, \underline{\mathcal{Y}}$ are singularly equivalent if there is a triangle equivalence $D_{sg}(\underline{\mathcal{X}}) \cong D_{sg}(\mathcal{Y})$.

The main purpose of this talk is to study the following question.

Question. Let \mathcal{A} be an abelian category with enough projective objects. Let \mathcal{X}, \mathcal{Y} be resolving subcategories of \mathcal{A} . When are the stable categories $\underline{\mathcal{X}}, \underline{\mathcal{Y}}$ singularly equivalent?

We give a sufficient condition for two stable resolving subcategories to be singularly equivalent. We also apply it to resolving subcategories of module categories of commutative Gorenstein rings, and characterize the simple hypersurface singularities of type A_1 in terms of singular equivalence classes.

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Mayu Tsukamoto (Osaka City University) Hochschild cohomology of q-Schur algebras

Let k be a field of characteristic zero and let $q \in k$ be a root of unity. Let (ρ, E) be the natural representation of a general linear group. Then GL(E) also acts on $E^{\otimes m}$ (tensor space of E) diagonally. Moreover a symmetric group on m-letters acts $E^{\otimes m}$ by place permutation. Then these actions commute. Thus $S(n,m) := \operatorname{End}_{k\mathfrak{S}_m}(E^{\otimes m})$ is called a Schur algebra. Its q-analogue is a q-Schur algebra. Namely, GL(E) is replaced by a quantum group $U_{q^{1/2}}(\mathfrak{gl}_n)$, the symmetric group is replaced by an Iwahori-Hecke algebra $\mathcal{H}_{q,m}$. Thus $S(n,m) := \operatorname{End}_{\mathcal{H}_{q,m}}(E^{\otimes m})$ is called a q-Schur algebra.

Let A be an algebra over k. We recall the definition of the *i*-th Hochschild cohomology group of A, $\operatorname{HH}^{i}(A) := \operatorname{Ext}_{A^{en}}^{i}(A, A)$, where $A^{en} := A \otimes_{k} A^{op}$ acts on the left on A by left and right multiplication. Then $\operatorname{HH}^{\bullet}(A) := \bigoplus_{i>0} \operatorname{HH}^{i}(A)$ is a graded

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algebra with the Yoneda product. It is known that $HH^{\bullet}(A)$ is a derived invariant.(cf. [1], [2])

In this talk, I will explain results on the calculation of the Hochschild cohomology group of q-Schur algebras. We calculate a Hochschild cohomology of an certain algebra. This follows the result of [3], the result of [4] and the fact that the Hochschild cohomology ring is a derived invariant.

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A. Umamaheswaran (Periyar University)

Covers and Envelopes of $\mathcal{U}_{\mathcal{X}}$ -injective modules

In this talk, we introduce the notions of $\mathcal{U}_{\mathcal{X}}$ -projective, $\mathcal{U}_{\mathcal{X}}$ -injective and $\mathcal{U}_{\mathcal{X}}$ -flat modules and give their characterizations, where $\mathcal{U}_{\mathcal{X}} = \mathcal{X}^{\perp}$. We prove that the class of all $\mathcal{U}_{\mathcal{X}}$ -projective modules is Kaplansky. Further, we prove that (i) every module has an $\mathcal{U}_{\mathcal{X}}$ -projective cover over a hereditary Noetherian ring and a \mathcal{X} -injective envelope over a hereditary ring R, (ii) every module has an $\mathcal{U}_{\mathcal{X}}$ -injective precover and an $\mathcal{U}_{\mathcal{X}}$ injective envelope over a hereditary ring. Finally, if R is a self $\mathcal{U}_{\mathcal{X}}$ -injective hereditary ring, then we show that a module M is $\mathcal{U}_{\mathcal{X}}$ -projective if and only if M is a direct sum of a projective module and a coreduced $\mathcal{U}_{\mathcal{X}}$ -projective module.

Jorge Vitória (Università degli Studi di Verona) Silting modules and silting complexes

Support τ -tilting theory for finite dimensional algebras extends classical tilting theory, allowing to complete the parametrisation of certain structures in the module category and in its derived category ([1]). The new concept of silting modules ([2]) provides an adequate setup for such parametrisations over arbitrary rings, while keeping some of the features of (possibly large) tilting modules. In this talk, we will define silting modules and discuss their relations with silting complexes and (co-)t-structures in the derived category. This is joint work with Lidia Angeleri Hügel and Frederik Marks.

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Y. V. Volkov (Saint-Petersburg State University) About derived equivalences of orbit categories

Let k be a commutative ring, \mathcal{A} and \mathcal{B} be two k-linear categories with an action of group G. We introduce the notion of standard G-equivalence from \mathcal{DB} to \mathcal{DA} . We construct several maps which connect the sets of standard equivalences and Gequivalences from \mathcal{DB} to \mathcal{DA} and the set of standard equivalences from $\mathcal{D}(\mathcal{B}/\mathcal{G})$ to $\mathcal{D}(\mathcal{A}/\mathcal{G})$, where we denote by \mathcal{A}/\mathcal{G} the orbit category of \mathcal{A} by G. These maps connect the derived Picard group of \mathcal{A} and \mathcal{A}/\mathcal{G} in the case where $\mathcal{A} = \mathcal{B}$. We investigate properties of these maps in the case where $\mathcal{A} = \mathcal{B} = \mathcal{R}$ is a Frobenius k-algebra and G is the cyclic group generated by its Nakayama automorphism ν . This is joint work with A. Zvonareva.

Cheng-Xi Wang (Beijing Normal University) A group structure of hyperbolic curve bundle

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This is joint work with Kai-Rui Wang.

Based on arithmetic of the hyperbolic curve modulo N

$$H(x^2 - Dy^2 \equiv 1(modN)),$$

where D is a non square integer and $gcd(D, N) = 1^{[1]}$, we extend it into a bigger hyperbolic curve group, called a hyperbolic curve bundle and denoted by $\mathbb{H}(D, N)$. It is given by

$$\mathbb{H}(D,N) = \bigcup_{(r_i,N)=1}^{\varphi(N)} H(x^2 - Dy^2 \equiv r_i(modN)).$$

We may verify that $\mathbb{H}(D, N)$ is a finite abelian group by the following operation: for $\forall (a, b), (a', b') \in \mathbb{H}(D, N),$

$$(a,b) \cdot (a',b') = (a+b\sqrt{D})(a'+b'\sqrt{D}) = a''+b''\sqrt{D} = (a'',b''),$$

where its unit element is (1,0), and if $(a,b) \in H(x^2 - Dy^2 \equiv r(modN))$ then its inverse $(a,b)^{-1} = (a'',-b'')(a',b')$, where (a'',b'') = (a',b')(a,b) and $(a',b') \in H(x^2 - Dy^2 \equiv r^{-1}(modN))$.

In fact, if each set $H_r = H(x^2 - Dy^2 \equiv r(modN))$ is regarded as a coset of the unit hyperbolic curve H_1 , that is, $H_r = (a, b)H_1$ for some $(a, b) \in H_r$, then its group index $[\mathbb{H} : H_1] = \varphi(N)$, and so its order $|\mathbb{H}(D, N)| = \varphi(N)|H_r|$, where $\varphi(N)$ is Euler's totient function and $|H_1| = |H_r|$ such as $|H_1| = p - (\frac{D}{p})$ for N taking odd prime p with $(\frac{\cdot}{p})$ Legendre symbol by computing Jacobi sum. Thus we introduce another form of it : Each $(a, b) \in \mathbb{H}(D, N)$ if and only if

$$(a,b)^{|\mathbb{H}(D,N)|} \equiv (1,0) (modN),$$

as a generalization of Euler's totient theorem, because of its quotient group $\mathbb{H}/H_1 \cong (\mathbb{Z}/N\mathbb{Z})^*$.

In addition, its applications are at least in the aspects of public key cryptography, primality testing, Hamiltonian graph, etc.

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Peter Webb (University of Minnesota)

Combinatorial restrictions on the AR quiver of a triangulated category

In a triangulated category with Auslander-Reiten triangles we use additive functions on the quiver (rather than on the tree class) to pin down the possible structures that can occur. We prove that if the quiver has Dynkin tree class then there is only one (shift-) component, thereby generalizing a theorem of Scherotzke, and by analogy with a theorem of Auslander for module categories. We also provide restrictions in the case of extended Dynkin class, and give restrictions on the position of objects in the quiver.

Sven-Ake Wegner (Bergische Universität Wuppertal) Exact couples in semi-abelian categories

Consider an exact couple in a semiabelian category in the sense of Palamodov, i.e., in an additive category in which every morphism has a kernel as well as a cokernel and the induced morphism between coimage and image is always monic and epic. Assume that the morphisms in the couple are strict, i.e., they induce isomorphisms between their corresponding coimages and images.

In the talk we show that the classical construction of Eckmann and Hilton in the situation explained above produces two derived couples which are connected by a natural bimorphism. The two couples correspond to the a priori distinct cohomology objects, the left resp. right cohomology, associated with the initial exact couple. The derivation process can be iterated under additional assumptions.

The results we explain in this talk are based on joint work with Ya. Kopylov (Sobolev Insitute of Mathematics, Novosibirsk, Russia).

Jiaqun Wei (Nanjing Normal University) Repetitive equivalences and Wakamatsu-tilting theory

Repetitive equivalences are more general than derived equivalences and Wakamatsutilting modules are more general than tilting modules. We show there are close relations between repetitive equivalences and Wakamatsu-tilting theory. We generalize Wakamatsu-tilting modules to Wakamatsu-tilting complexes (more further to Wakamatsu-silting complexes). Characterizations of Wakamatsu-silting complexes are given.

Paweł Wiśniewski (Nicolaus Copernicus University) Algebras of generalized standard semiregular type

This is report on joint work with A. Skowroński.

Let A be an algebra over an algebraically closed field K, modA the category of finitely generated right A-modules, and Γ_A the Auslander-Reiten quiver of A. A component \mathcal{C} of Γ_A is called semiregular if \mathcal{C} does not contain both projective module or an injective module. Moreover, let $\operatorname{rad}_A^\infty$ be the infinite radical of modA, that is, the intersection of all powers $\operatorname{rad}_A^i, i \geq 1$ of the Jacobson radical rad_A of modA. Following Skowroński [S], a component \mathcal{C} of Γ_A is said to be generalized standard if $\operatorname{rad}_A^\infty(X,Y) = 0$ for all modules X and Y in \mathcal{C} .

An interesting open problem concerns description of basic, indecomposable algebras A for which Γ_A consists of semiregular components. Prominent classes of such algebras are formed by the representation-infinite hereditary algebras and quasitilted algebras of canonical type. We refer to the recent articles [BS], [BSSW], [JS] for some result concerning the structure of tame algebras with semiregular Auslander-Reiten components.

The aim of the talk is to present a complete description of basic, indecomposable algebras A for which all components of Γ_A are semiregular and generalized standard. A crucial role in this description will be played by coherent sequences of tame quasitilted algebras of canonical type.

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Fan Xu (Tsinghua University) From Hall algebras to cluster algebras

In this talk, we present the constructions of Hall algebras of some orbit triangulated categories via derived Hall algebras for derived categories and characterize relations between them. As an application, we construct a natural algebra homomorphism from the Ringel-Hall algebra of a hereditary algebra to the corresponding quantum cluster algebra. This gives alternative proofs of some results recently obtained by Berensein and Rupel. Partial results in the talk are based on joint work with Ming Ding.

Dong Yang (Nanjing University)

Algebraic stratification and derived simplicity

Recollements and stratifications of triangulated categories were introduced by Beilinson, Bernstein and Deligne to give a categorical framework for the construction of perverse sheaves on stratified spaces. In this talk I will focus on derived module categories of finite-dimensional algebras. In this context recollements and stratifications are interesting because the computation of various homological invariants can be reduced via recollements and stratifications to derived simple algebras (i.e. algebras whose derived module category admits no non-trivial recollements). I will talk about the following three problems:

- (1) the relationship between ring-theoretical stratification and stratification of the derived module category;
- (2) the comparison of derived simplicities with respect to various (unbounded, bounded, left bounded,...) derived categories;
- (3) the validity/failure of a derived version of the Jordan–Hoelder property.

Guiyu Yang (Shandong University of Technology) Indecomposable projective modules of 0-Schur algebras

We will explain how to construct a complete set of indecomposable projective modules for the 0-Schur algebras following the geometric construction of 0-Schur algebras given by B. T. Jensen and X. Su. We will also give a short introduction of irreducible maps between the indecomposable projective modules. Then we will use these results to construct the Gabriel quiver and relations for the 0-Hecke algebra $H_0(5)$. This is based on joint work with B. T. Jensen and X. Su.

Yichao Yang (Zhejiang University)

On representation rings in the context of monoidal categories

In general, representation rings are well-known as Green rings from module categories of Hopf algebras.

In this paper, we study Green rings in the context of monoidal categories such that representations of Hopf algebras can be investigated through Green rings of various levels from module categories to derived categories in the unified view-point. Firstly, as analogue of representation rings of Hopf algebras, we set up the so-called Green rings of monoidal categories, and then list some such categories including module categories, complex categories, homotopy categories, derived categories and (derived) shift categories, etc. and the relationship among their corresponding Green rings.

The main part of this paper is to characterize representation rings and derived rings of a class of finite dimensional Hopf algebras constructed from covering quiver and Hopf ideals, that is, the Nakayama truncated algebras KZ_n/J^d under some constraints on K, n and d. For the representation ring $r(KZ_n/J^d)$, we completely determine its generators and the relations of generators via the method of Pascal triangle. For the derived ring $dr(KZ_n/J^2)$ (i.e., d = 2), we determine its generators and give the relations of generators. In these two aspects, the polynomial characterizations of the representation ring and the derived ring are both given. This is joint work with Min Huang and Fang Li.

Chang Ye (Zhejiang University)

Gorenstein projective modules over a class of generalized matrix algebras and their applications

In this article, we introduce a class of generalized matrix algebras, in which each algebra is called a normally upper triangular gm algebra, and characterise Gorenstein projective modules over this class of algebras. The importance of normally upper triangular gm algebras for us is that it includes the so-called *path algebra of a quiver* over an algebra and generalized path algebra. Due to this, we characterize Gorenstein projective modules over path algebras of quivers over algebras and generalized path algebras. The algebras and generalized path algebras. The algebras are constructed by the result on generalized path algebras. This is joint work with Fang Li.

Yingbo Zhang (Beijing Normal University)

Algebras with homogeneous module category are tame

This is joint work with Xu Yunge. The celebrated Drozd's theorem asserts that a finite-dimensional basic algebra Λ over an algebraically closed field k is either tame or wild, whereas the Crawley-Boevey's theorem states that given a tame algebra Λ and a dimension d, all but finitely many isomorphism classes of indecomposable Λ -modules of dimension d are isomorphic to their Auslander-Reiten translations and hence belong to homogeneous tubes. In this paper, we prove the inverse of Crawley-Boevey's theorem, which gives an internal description of tameness in terms of Auslander-Reiten quivers. (arXiv:1407.7576)

Pu Zhang (Shanghai Jiao Tong University) Gorenstein singularity category

We define the Gorenstein singularity category. It is a Gorenstein version of the stabilized derived category in the sense of R.-O.Buchweitz, or of the singularity category in the sense of D. Orlov. With this we will give an alternative description of the Gorenstein defect category in the sense of P. A. Bergh, D. A. Jorgensen, and S. Oppermann.

Minghui Zhao (Beijing Forestry University) Geometric realizations of Lusztig's symmetries

Let **f** be the positive part of the quantum group **U** corresponding to a quiver Q = (I, H). In [2], Lusztig gave a geometric realization of **f** as the Grothendieck group of the category $\mathcal{Q}_{\mathbf{V}}$ of perverse sheaves on the variety $E_{\mathbf{V}}$ of representations of Q. Lusztig also introduced some symmetries T_i on **U** for all $i \in I$. Since $T_i(\mathbf{f})$ is not contained in **f**, Lusztig introduced two subalgebras $_i\mathbf{f}$ and $^i\mathbf{f}$ of **f** for any $i \in I$, where $_i\mathbf{f} = \{x \in \mathbf{f} \mid T_i(x) \in \mathbf{f}\}$ and $^i\mathbf{f} = \{x \in \mathbf{f} \mid T_i^{-1}(x) \in \mathbf{f}\}$. In case $i \in I$ is a sink (resp. source) of Q, $_i\mathbf{f}$ (resp. $^i\mathbf{f}$) can be realized as the Grothendieck group of the category $_i\mathcal{Q}_{\mathbf{V}}$ (resp. $^i\mathcal{Q}_{\mathbf{V}}$) of perverse sheaves on a subvariety $_iE_{\mathbf{V}}$ (resp. $^iE_{\mathbf{V}}$) of $E_{\mathbf{V}}$.

Assume that $i \in I$ is a sink of Q. Let Q' be the quiver by reversing the directions of all arrows in Q containing i. Consider two I-graded vector spaces \mathbf{V} and \mathbf{V}' such that $\underline{\dim}\mathbf{V}' = s_i(\underline{\dim}\mathbf{V})$. In the case of finite type, Kato introduced an equivalence of ${}_i\mathcal{Q}_{\mathbf{V},Q}$ and ${}^i\mathcal{Q}_{\mathbf{V}',Q'}$ and study the properties of this equivalence in [1]. In my talk, we generalize his constriction to arbitrary cases and prove that the map induced by this equivalence realizes Lusztig's symmetry $T_i :_i \mathbf{f} \to^i \mathbf{f}$. We also study some properties of T_i in the sense of 'geometrization'.

References

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Guodong Zhou (East China Normal University) Koszul duality and Batalin-Vilkovisky structure

Analogous to a recent result of N. Kowalzig and U. Kraehmer for twisted Calabi-Yau algebras, we show that the Hochschild cohomology ring of a Frobenius algebra with semisimple Nakayama automorphism is a Batalin-Vilkovisky algebra, thus generalizing a result of T.Tradler for symmetric algebras. It is well-known that for a Koszul algebra which is twisted Calabi-Yau with semisimple Nakayama automorphism, its Koszul dual is a Frobenius algebra with semisimple Nakayama automorphism. We show that the Hochschild cohomology ring of this algebra and that of its Koszul dual are isomorphic as Batalin-Vilkovisky algebras. This confirms a conjecture of R. Rouquier.

Yu Zhou (Bielefeld University)

Intersection-dimension formulae in marked surfaces

For a triangulation of a marked surface, there is an associated quiver with potential (Q, W) constructed by Fomin-Shapiro-Thurston and Labardini. Through the Ginzburg algebra $\Gamma = \Gamma(Q, W)$ of (Q, W), by Amiot's work, there are three triangulated categories: the perfect derived category per (Γ) , the finite dimensional derived category of $\mathcal{D}_{fd}(\Gamma)$, and the cluster category of $\mathcal{C}(\Gamma)$. In this talk, we give some formulae in these categories which connect intersections of curves in the surface and dimensions of certain or total Hom spaces of objects in the categories. Some applications of these formulae are included. These are joint works with Yu Qiu.

Alexandra Zvonareva (Saint Petersburg State University) On the derived Picard group of Nakayama algebras

The derived Picard group of an algebra A is the group of isomorphism classes of two-sided tilting complexes in $D^b(A \otimes A^{op})$ with the product of the classes of X and Y given by the class $X \otimes_A Y$. In my talk I will discuss how to use the technique of silting mutations developed by Aihara and Iyama to find the generating set of the derived Picard group of a Brauer star algebra and how these results can be extended to any Nakayama algebra by a joint work with Yury Volkov.

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