

Titles and Abstracts for RACMA

1. Yves Bourgault, University of Ottawa

Title: Numerical Modelling in Cardiac Electrophysiology

Abstract: Nowadays mathematical models are used to study the propagation of electrical waves in the heart and understand cardiac pathologies, such as arrhythmias, ventricular fibrillations, etc. These electrophysiological waves are usually modelled using reaction-diffusion partial differential equations (PDE) for the propagation in the myocardial tissue coupled with a system of stiff ordinary differential equations (ODE) representing the ionic activity at the cell level. The highly nonlinear nature of the system together with the complex geometry of the heart makes it essential to use efficient numerical methods for its solution. We will briefly present the mathematical model for cardiac propagation and introduce numerical methods to solve such equations. In particular, anisotropic mesh adaptation and time-stepping methods will be covered, and their performance will be illustrated with numerical test cases. We will also explain how we obtained a very detailed geometrical model of the heart from medical images, and how propagating electrical waves can be computed on such geometrical model.

2. Shuhua Chang, Tianjin University of Finance and Economics

Title: A Front-fixing Finite Element Method for the Valuation of American Options with Regime Switching

Abstract: American option pricing problems under regime-switching model are considered in this talk. The conjectures about the position of the early exercise prices are proved, which generalize the results in Yi's paper (2008) by allowing the interest rates to be different in two states. A front-fixing finite element method for the free boundary problems is proposed and implemented. Its stability is established under reasonable assumptions. Numerical results are given to examine the rate of convergence of our method and compare it with the usual finite element method.

3. Wenbin Chen, Fudan University, China

Title: NUMERICAL SIMULATIONS FOR SOME NEW EPITAXIAL THIN FILM MODELS

Abstract: In 2006, Evans, Thiel and Bartelt proposed one new continuum evolution equations to model the growth of the thin films. In this talk, we will discuss the energy structure of the model, and convex-concave splitting method is applied to derive their linear numerical schemes, by treating the convex part implicitly and the concave part explicitly. We present the properties of our numerical schemes such as the growth rate estimates of physically quantity named as the surface roughness, unconditional solvability and stability, local-in-time convergence, and longtime behavior of the schemes. The collaborative work is finished with Zhenghau Chen, Jin Cheng, Yuan Liu and Xiaoming Wang.

4. **Zhangxing(John) Chen, University of Calgary, Canada**

Title: How Parallel Simulators Recover More Oil?

Abstract. For large oil and gas heterogeneous fields, vast amounts of seismic, geologic and dynamic reservoir data yield high-resolution geological models. These models involve mega (million) to giga (billion) grid block cells. Recent advances in the development of a general parallel reservoir simulator platform are capable of the solution of field simulations of these sizes within minutes. These parallel simulations provide increased recovery of oil and gas resources, due to a full utilization of available data and a better understanding of the chemical and physical mechanisms involved, process design and uncertainty analysis. This talk will present these advances in the development of a general parallel reservoir simulator platform on CPUs and GPUs. Applications to the black oil, compositional and thermal simulators will be addressed.

5. **Junzhi Cui, University of Nevada, Las Vegas USA**

Title: The Atomic-Continuum Coupled Model at Micro- Nano-Scales for the Thermo-Mechanical Performances of Metallic Materials

Abstract: In order to study the thermo-mechanical behaviors of metallic devices in micro- nano-scales, a Thermo-Mechanics Atomic-Continuum Coupled (TM-ACC) model and its algorithm are presented in this paper. And then the thermo-dynamics properties of metallic materials are numerically simulated. First by means of the consistent hypothesis of deformation, the structural deformation of atomic clusters is related with continuum one. Accordingly, the free energy density are calculated by introducing Representative Volume Element (RVE), and it couples the physical parameters of different model, and there is no transition areas between atom and continuum region, so it avoids difficult interface problem. Then to construct basic deformation elements and deduce mechanical tensors, the primitive cells of the complex Bravais lattice is used instead of lattice partition. Under quasi-harmonic approximation, the expressions the free energy density and thermo-mechanical parameters are derived. And then the thermo-mechanical coupled equations under micro-nano-meter scales are obtained by the conservation of momentum, energy and the second law of thermo-dynamics. Finally, numerical simulations of single-crystal nano-wire are implemented. And the results show the in-homogeneity of strain and stress tensor inside single-crystal nano-wire after deformation. It verifies the rationality of TM-ACC model, TM-ACC model can be used to predict the thermo-dynamic properties under different temperature and loads, for those the physical experiment is very difficult.

6. **Ryan Fernandes , Bernard Bialecki , Graeme Fairweather**

Title: An ADI orthogonal spline collocation method for nonlinear reaction-diffusion systems on evolving domains

Abstract: Since their introduction over fifty years ago, ADI methods have been employed effectively for the time stepping in the numerical solution of a variety

of time-dependent multidimensional problems. Their primary attraction is that they reduce such a problem to independent systems of one-dimensional problems. On the other hand, orthogonal spline collocation (OSC) has evolved as a valuable technique for the spatial discretization of several types of differential equations. Their popularity is due in part to their conceptual simplicity, wide applicability and ease of implementation. A well-known advantage of OSC methods over finite element Galerkin methods is that the calculation of the coefficients in the equations determining the approximate solution is very fast, since no integrals need to be evaluated or approximated. Another attractive feature of OSC methods is their superconvergence properties. In this talk, we describe an ADI method for the solution of a class of two-component nonlinear reaction-diffusion problems on evolving domains. In our new method, OSC is used for the spatial discretization, while the time-stepping is done with an algebraically linear ADI method based on an extrapolated Crank-Nicolson OSC method. The ADI OSC method is efficient, requiring at each time level only $O(N)$ operations where N is the number of unknowns. The results of numerical experiments involving well-known examples of reaction-diffusion systems from the literature demonstrate its efficacy.

7. **Benqi Guo, Department of Mathematics University of Manitoba**

Title: Recent Progress and Challenging Issues on Algebraic Approximation and Approximation Theory of the p and h - p Finite Element method

Abstract: In recent decades the algebraic approximation theory and the approximation theory of the h and h - p finite element methods have made substantial progresses. Among all developments two are fundamental. The first one is establishment of algebraic approximation theory in the Jacobi-weighted Besov and Sobolev spaces, and the second one is construction of the local Jacobi-weighted projection-interpolation which possesses optimality, locality, conformity and uniformity. These breakthroughs lay down the mathematical foundation of direct and inverse algebraic approximation theory and lead to the optimal convergence of the h and h - p finite element methods for problems on smooth and unsmooth domains, and answer two unresolved fundamental issues in the past many decades, i.e., what are the most appropriate function spaces for algebraic approximation and how to construct a continuous and piecewise polynomial possessing optimal error bound locally and globally for singular functions satisfying homogeneous and non-homogeneous Dirichlet boundary condition on general meshes with elements of all types.

In this talk we will present the major results on algebraic approximation theory and the approximation theory of the h and h - p finite element methods, including concepts, methodology and significance.

8. **Bin Han, University of Alberta, Edmonton, Alberta, Canada**

Title: Applications of Directional Complex Tight Framelets to Image Processing

Abstract: Real-valued separable wavelets and framelets are known to have some shortcomings, in particular, they lack directionality for high dimensional problems such as image and video processing. In this talk, we show that directionality can be greatly improved by using complex tight framelets. We first introduce directional complex tight framelets and then we explore several exciting applications of directional complex tight framelets to image denoising and image inpainting. Numerical algorithms will be developed and we shall show that directional complex tight framelets have superior performance compared with many other transform-based methods.

9. **Huaxiong Huang, York University**

Title: An immersed boundary method for mass transfer across permeable moving interfaces

Abstract: In this talk, we present an immersed boundary method for mass transfer across permeable deformable moving interfaces interacting with the surrounding fluids. One of the key features of our method is the introduction of the mass flux as an independent variable, governed by a non-standard vector transport equation. The flux equation, coupled with the mass transport and the fluid flow equations, allows for a natural implementation of an immersed boundary algorithm when the flux across the interfaces is proportional to the jump in concentration. As an example, the oxygen transfer from red blood cells in a capillary vessel is used to illustrate the applicability of the proposed method. We show that our method is capable of handling multi-physics problems involving fluid-structure interaction with multiple deformable moving interfaces and (interfacial) mass transfer simultaneously. This is joint work with Xiaobo Gong and Zhaoxin Gong, Shanghai Jiaotong University.

10. **Hong Jiang, Bell Labs, Murray Hill, NJ, USA**

Title: Constrained and Preconditioned Stochastic Gradient Method

Abstract: In this talk, we consider stochastic approximations which arise from such applications as data communications and image processing. We demonstrate why constraints are needed in a stochastic approximation and how a constrained approximation can be incorporated into a preconditioning technique to derive the preconditioned stochastic gradient method (PSGM). We perform convergence analysis to show that the PSGM converges to the theoretical best approximation under some simple assumptions on the preconditioner and on the independence of samples drawn from a stochastic process. Simulation results are presented to demonstrate the effectiveness of the constrained and preconditioned stochastic gradient method.

11. **Buyang Li and Weiwei Sun, Department of Mathematics, Nanjing University & Department of Mathematics, City University of Hong Kong**

Title: Error analysis of linearized semi-implicit finite element methods for nonlinear parabolic equations without grid-ratio conditions

Abstract: Due to the nonlinear structure, traditional approach for error estimates of fully discrete finite element methods for nonlinear parabolic equations often require certain restrictions on the grid ratio (between the time-step size and spatial mesh size), not only for explicit time-stepping schemes but also for implicit or semi-implicit schemes. We introduce a new approach to analyze the discretization errors, which decouples the errors from the temporal and spatial directions and avoid the restrictive conditions on the grid ratio. Here we discuss recent advance of this new approach.

12. **Jichun Li, University of Nevada Las Vegas, USA.**

Title: Mathematical analysis and finite element modeling of invisibility cloaks with metamaterials

Abstract: Since the pioneering works of Pendry et al and Leonhardt in 2006 (both published in Vol.312 of Science, June 23, 2006), many interesting works on cloaks with metamaterials have been published. In this talk, I'll focus on some cloak models and study them from the mathematical and simulation point of view. Both frequency and time-domain models and simulation results will be presented.

13. **Kaitai Li, School of Mathematics and Statistics, Xi'an Jiaotong University**

Title: A Finite Element Dimensional Splitting Methods for Rotating Navier-Stokes Equations and Applications to Geometric Shape Control of the Blade in the Impeller

Abstract: The Drag Functional (Hydrodynamical Force acting on the Boundary) is chosen as objective functional for shape optimization of Navier-Stokes boundary. Since conjugate gradient methods to compute optimization must do numerical differential for 3D Stress tensor and Gateaux derivative of solutions of Navier-Stokes equation with respect to the shape of boundary. Thus is a difficult and no efficiently problem.

Our contributions are that all computation for conjugate gradient method for this kind of optimization do not need numerical differentiation for stress tensor and Gateaux derivative of solutions of Navier-Stokes equation with respect to the shape of boundary it is only to solve two dimensional boundary layer equations. This method is based on "A Finite Element Splitting Method for Rotating Navier-Stokes Equations".

As well known that governing equations in the Channel in Turbomachinery are 3D Rotating Navier-Stokes Equations. Since the boundary geometry is very complex it is difficulty to do efficiently numerical simulation. This method is applying the boundary geometry to establish a called dimensional splitting method including one dimensional problem and a series of two dimensional problem on a two dimensional manifold which is called 2D-3C Navier-Stokes equations. This method do not generating 3D mesh and a bi-parallell algorithm will be established.

14. **Bernadette Miara, Universit'e Paris-Est**

Title: Stability of the Contact Zone for a Shallowshell

Abstract: This work deals with the variation of the solution to an obstacle problem with respect to the variation of its parameters. More precisely, a mechanical structure is pushed by some external forces against an obstacle in such a way that the equilibrium solution involves a part of the domain in which the structure is strictly in contact with the obstacle. It is known from the theory of variational inequalities that studying the variation of the solution as the external forces vary amounts to studying the variation of the boundary of this contact zone. This problem has been studied in previous works in the scalar case, and it was open in the general case where the unknown is a vector field, due to the coupling between the components. As a first step, the present work considers the case of a linearly elastic shallow membrane shell where the coupling between the in-plane and normal components of the displacement arises from the curvature.

This is a joint work with Dr A.L'eger, Laboratoire de M'ecanique et d'Acoustique, Marseille.

Ref:

- (i) Schaeffer D.G., A stability theorem for the obstacle problem, *Advances in mathematics* 16, 34-47, 1975.
- (ii) L'eger A., Miara B., (2008), "Mathematical justification of the obstacle problem in the case of a shallow shell", *J. Elasticity*, 90, 241-257.
- (iii) L'eger A., Miara B., (2008), "The obstacle problem for shallow shells: a curvilinear approach", *Int. J. Numerical Analysis and Modeling, Series B*, Vol. 2, (1), 1-26, 2011
- (iv) L'eger A., Miara B., (2014), 'Stability in the obstacle problem for a shallow shell', to appear in *Analysis and Applications*

15. **P. D. Minev and J.L. Guermond**

Title: High-order Artificial Compressibility for the Navier-Stokes Equations

Abstract: We introduce a generalization of the artificial compressibility method for approximation of the incompressible Navier-Stokes equations. It allows for the construction of schemes of any order in time that require the solution of a fixed number of vectorial parabolic problems, depending only on the desired order of the scheme. These problems have a condition number that scales like $O(\delta t h^{-2})$, with δt being the time step and h being the spatial grid size. This approach has several advantages in comparison to the traditional projection schemes widely used for the unsteady Navier-Stokes equations. First, it allows for the construction of schemes of any order for both, the velocity and pressure, while the best proven accuracy achievable by a projection scheme is second order on the velocity and $3/2$ order on the pressure. Second, the projection schemes require the solution of an elliptic scalar problem for the pressure that has a condition number $O(h^{-2})$, in addition to a vectorial parabolic problem for the velocity. This makes them slower if iterative methods are used to solve the linear systems. Third, the artificial compressibility schemes of second or higher order

based on a defect correction approach presented below have superior stability properties as compared to a second order projection scheme. Finally, the ability to construct higher order schemes allows to incorporate time step control techniques which greatly improves their efficiency.

16. Hongxing Rui, Shandong University

Title: Cell Centered Finite Difference Methods for Darcy-Forchheimer Model in Porous Media

Abstract: Darcy-Forchheimer model is a nonlinear model to describe flow which cannot be modeled by Darcys law in porous media. In this talk we will present some mixed element methods and cell-centered finite difference methods based on the lowest order mixed finite element method to non-Darcy (Darcy-Forchheimer) flow problems. We will give the approximate schemes, existence and uniqueness analysis, error estimate and numerical examples.

17. Sivabal Sivaloganathan, University of Waterloo, Canada

Title: Mathematical & Computational Challenges in the Biomedical Sciences

Abstract: Whilst the application of mathematical and computational methods to the biomedical sciences is not new, in recent years there has been a dramatic growth in the interaction between mathematical and biomedical scientists. In this talk, I will highlight some current problems in neurosurgery and clinical oncology and the potential for mathematical/computational scientists to make significant contributions in these fields.

18. Petr N. Vabishchevich, Nuclear Safety Institute, Russian Academy of Sciences

Title: Numerical solving the boundary value problem for fractional powers of elliptic operators

Abstract: Nowadays, non-local applied mathematical models based on the use of fractional derivatives in time and space are actively discussed. An interesting example is a boundary value problem for a fractional power of an elliptic operator. For example, in a bounded domain Ω , we search the solution of the problem

$$\begin{aligned} (-\Delta)^\alpha u &= f(x), \quad x \in \Omega \\ u(x) &= 0, \quad x \in \partial\Omega \end{aligned}$$

where $0 < \alpha < 1$.

Different approaches are employed to solve numerically such boundary value problems. The simplest variant is associated with the explicit construction of the solution using the known eigenvalues and eigenfunctions of the elliptic operator with diagonalization of the corresponding matrix. Another approach is based on the representation of an elliptic operator power in the form of a contour

integral (the Dunford-Cauchy representation) and the application of appropriate quadrature formulas with nodes of integration on the complex plane. It is also necessary to note the possibility of finding the solution for the fractional power of an elliptic operator as the solution of an elliptic boundary value problem of higher dimension, i.e., we introduce a new variable.

After constructing finite difference or finite element approximations, from the boundary value problem for the fractional power of the elliptic operator, we arrive at the problem of multiplication of the fractional power of the matrix corresponding to elliptic operator by the vector, which corresponds to the right-hand side. For such a matrix problem, different approaches are used: Krylov subspace methods, contour integration and so on. Special attention should be given to the methods that solve the Cauchy problem for the corresponding evolutionary equation.

For solving the boundary value problem for a fractional power of an elliptic operator, we use the transition to a pseudo-parabolic equation. The stability of the two-level scheme with weights is shown. Numerical experiments for a model two-dimensional problem are performed using the standard finite element approximations.

19. Frederic Y. M. Wan, Department of Mathematics, University of California, Irvine

Title: Robust Biological Developments

Abstract: The patterning of many developing tissues is orchestrated by gradients of morphogens through a variety of elaborate regulatory interactions. Such interactions are thought to make gradients robust—i.e. resistant to change in the face of genetic or environmental perturbations. Just how this might be done is a major unanswered question. Past numerical simulations and analytical studies suggest that robustness of signaling gradients cannot be attained by negative feedback (of the Hill's function type) on signaling receptors but can be achieved through morphogen degradation by non-signaling receptor molecules (or non-receptors for short) such as heparan sulfate proteoglycans. However, empirical evidence of feedback regulating signaling gradients has been reported in the literature. The present paper undertakes a different approach to the role of feedback in robust signaling gradients (and therewith robust biological developments). The overall goal of the project is to investigate the effectiveness of feedback on ligand synthesis, receptor-mediated degradation, non-receptor synthesis and other regulatory processes in morphogen gradient systems. As a first step, we present in this talk a proof of concept examination of a new approach to feedback processes that would lead to robust development. The research is supported in part by NIH Grants R01GM067247, and P50-GM076516. The R01 was awarded through the Joint NSF/ NIGMS Initiative to Support Research in the Area of Mathematical Biology.

20. Kun Wang, College of Mathematics and Statistics, Chongqing University

Title: Efficient Finite Difference Methods for Helmholtz Equation with High Wave Numbers

Abstract: In this talk, new finite difference schemes are presented for solving the Helmholtz equation with high wave numbers in the polar and spherical coordinates. Firstly, new schemes which are pollution free are studied on some special domains. Let h denote the step size, it will be demonstrated that when solving the Helmholtz equation at large wave numbers and considering kh is fixed, the errors of the proposed new schemes decrease as h decreases even when k is increasing. Then, for general domains, coupled schemes are considered in which the pollution effect only happens in the neighborhood of zero, and it is pollution free in the rest part of the domain, too. Therefore, lots of computational cost will be reduced applying the new schemes. Numerical experiments will be shown to verify the superior of the developed methods. This is a joint work with Professor Yau Shu Wong and Doctor Jian Deng of University of Alberta.

21. **Liwei Xu, College of Mathematics and Statistics, Chongqing University, Institute of Computing and Data Sciences, Chongqing University**

Title: On Two Numerical Models of Nonlinear Water Wave Equations

Abstract: In this talk, we first introduce two numerical models, the Zakharov-Craig-Sulem model and the modified Green-Naghdi model, describing the propagation of nonlinear water waves. Then we discuss the design of corresponding numerical methods, spectral methods and discontinuous Galerkin methods, to solve these equations. Numerical solutions will be presented to show the efficiency and accuracy of both numerical models and methods. This is a joint work with Prof. Philippe Guyenne at UD, Prof. Fengyan Li at RPI, Dr. Maojun Li at CQU.

22. **Ningning Yan, Institute of System Sciences, Academy of Mathematics and System Science, Chinese Academy of Sciences**

Title: A multilevel correction method for optimal controls of elliptic equation

Abstract: In this talk we propose a multilevel correction method to solve optimal control problems constrained by elliptic equations with the finite element method. In the scheme, solving optimization problem on the finest finite element space is transformed to a series of solutions of linear boundary value problems by the multigrid method on multilevel meshes and a series of solutions of optimization problems on the coarsest finite element space. Our proposed scheme, instead of solving a large scale optimization problem in the finest finite element space, solves only a series of linear boundary value problems and the optimization problems in a very low dimensional finite element space, and thus can improve the overall efficiency for the solution of optimal control problems governed by PDEs.

23. **Xiu Ye, University of Arkansas at Little Rock**

Title: Recent Development of Weak Galerkin Methods

Abstract: The Weak Galerkin method is an extension of the standard Galerkin finite element method where classical derivatives were substituted by weakly defined derivatives on functions with discontinuity. Recent development of weak Galerkin methods will be discussed in the presentation.

24. **Zhimin Zhang, Beijing Computational Science Research Center and Wayne State University**

Title: How Many PDE Numerical Eigenvalues can We Trust?

Abstract: When approximating PDE eigenvalue problems by numerical methods such as finite difference and finite element methods, it is common knowledge that only a small portion of numerical eigenvalues are reliable. However, this knowledge is only qualitative rather than quantitative in the literature. Here we investigate the number of "trusted" eigenvalues by the finite element approximation of $2m$ -th order elliptic PDE eigenvalue problems. Our two model problems are the Laplace and bi-harmonic operators. We show that the number of reliable numerical eigenvalues can be estimated in terms of the total degree of freedom N of resulting discrete systems. The result is worse than what we used to believe in that the percentage of reliable eigenvalues with a pre-set convergent rate (also in terms of N) decreases with an increased N .

25. **Qingsong Zou, Sun Yat-Sen University, China**

Title: Analysis of High-order Finite Volume Methods

Abstract: In this talk, we will report our recent investigations on the fundamental mathematical theory of high order finite volume methods. The theory includes the stability, convergence and superconvergence analysis of a family of arbitrary order finite volume schemes on arbitrary quadrilateral meshes. A brand new from the trial to test space mapping plays a critical role in our analysis. Several numerical experiments will also be presented to support our theoretical findings.