Titles and abstracts for Group Action Forum

1. **Alejandro Adem, University of British Columbia**
   Title: Homotopy Group Actions and Group Cohomology
   Abstract: Let *G* be a finite group, in this talk we will discuss the notion of a homotopy action of *G* on a finite complex *X*. We will describe some natural cohomological invitations associated to this and how interesting geometric actions can arise. Examples and related work involving actions on spheres and their products will be described.

2. **Martin Bridgeman, Boston College**
   Title: The Isometry group of the Quasifuchsian Intersection Function
   Abstract: We consider the intersection function $I: QF(S) \times QF(S) \to R$ for quasifuch-sian space. We prove that the Isometry group is the extended mapping class group os the surface *S*.

3. **Michel Brion, Université Joseph Fourier, France**
   Title: Linearization of line bundles
   Abstract: Linearization of line bundles in the presence of algebraic group actions is a basic notion of geometric invariant theory, introduced by Mumford in [4]. Given an algebraic variety X over an algebraically closed field k, an action of an algebraic group G on X, and a line bundle L on X, a Glinearization of L is an action of G on the total space of L which lifts the Gaction on X and which is linear on fibers (i.e., the map $L_x \to L_{g \cdot x}$ is linear for any $g \in G$ and $x \in X$). If L is G-linearized, then so are its tensor powers $L^{\otimes n}$, where $n \in Z$, and hence G acts linearly on the spaces of global sections $\Gamma(X,L^{\otimes n})$ and on the graded algebra $\bigoplus_n \Gamma(X,L^{\otimes n})$; this is the starting point of the construction of quotients in geometric invariant theory. Also, when X is the total space of a principal G-bundle $X \to Y$, the G-linearized line bundles on X are exactly the pullbacks of line bundles on Y . On an arbitrary G-variety X, the G-linearized line bundles can be identified with the line bundles on the quotient stack [X/G].
   The proposed talk will discuss the existence of G-linearizations of a given line bundle (or of some tensor power) on a G-variety X. When G is connected linear and X is normal, it is known that some positive power $L^{\otimes n}$ is G-linearizable (see Mumford [4] for X complete, and Sumihiro [5, 6] for the general case). This result does not extend to nonnormal varieties, a classical example being the rational nodal curve X obtained from the projective line P^1 by identifying 0 and ∞: the natural action of the multiplicative group G_m on P^1 yields an action on X, and the G_m - linearizable line bundles on X are those of degree 0. To see this, consider the normalization map $\eta : P^1 \to X$. Then for any line bundle L on X, the pullback $\eta^*(L)$ is equipped with an isomorphism $\eta^*(L)_0 \cong \eta^*(L)_\infty$; if L is G_m -linearized, then this isomorphism is equivariant for the linear actions of G_m on both fibers. On the other hand, one checks that the weights of the G_m -actions on $O_{P^1}(n)_0$ and $O_{P^1}(n)_\infty$ differ by n, for any linearization of O_P

1 (n). Yet every line bundle on the above curve X admits a linearization after pullback under the Z-cover π : Y → X, where Y is an chain of projective lines $(L_n)_{n \in Z}$, and each L_n is glued to L_{n-1} and L_{n+1} at distinct marked points. Here Z acts on Y by translations, and G_m acts by its natural action on each L_n fixing both marked points. The following result yields a common generalization of Sumihiro's theorem and the above example:
Theorem. Let X be a seminormal variety equipped with an action of a connected linear algebraic group G. Denote by b G the group of multiplicative characters of G (this is a free abelian group of finite rank). Then there exists a b G-cover π : Y → X and a positive integer n such that $\pi^*(L^{\otimes n})$ is G-linearizable for any line bundle L on X.

4. **Georgios Daskalopoulos, Brown University**
   Title: Rigidities of Teichmulcer space
   Abstract: I will discuss applications of harmonic maps to solve rigidity questions of Teichmulcer space. This is joint work with C. Mese and builds further on the fundamental work of Gromov-Schoen and Jost-Yau.

5. **Haibao Duan, Institute of Mathematics,Chinese Academy of Sciences**
   Title: Schubert calculus and cohomology of Lie groups
   Abstract: The problem of determining the cohomology of a Lie group G was raised by E. Cartan in 1929. It has been a focus of algebraic topology due to the fundamental roles of Lie groups playing in geometry and topology. On the other hand Schubert calculus begun with the intersection theory of the 19 century. Clarifying this calculus had been a major theme of the 20 century algebraic geometry. We bring a connection between these two topics both with distinguished historical backgrounds, and demonstrate how Schubert calculus is extended as to give an explicit and unified construction of the integral cohomology ring $H^*(G)$ of all compact and 1—connected Lie groups G.

6. **Sorin Dumitrescu, Université Nice-Sophia Antipolis**
   Title: Quasihomogeneous real and complex geometric structures
   Abstract: This talk deals with rigid geometric structures which are quasihomogeneous, in the sense that they are locally homogeneous on an open dense subset of a manifold, but not on the all manifold. Our motivation comes from Gromov's open-dense orbit theorem and its application to prove the differential rigidity of some smooth Anosov systems. More precisely, we will present the classification of quasihomogeneous real analytic connections on surfaces (collaboration with A. Guillot) and the case of real analytic Lorentz metrics on threefolds (collaboration with K. Melnick). We will also present the corresponding classification results on complex manifolds.

7. **David Gabai, Princeton University**
   Title: On the classification of Heegarrd Splittings

Abstract: A longstanding problem is for a closed 3-manifild to list without duplication the irreducible Heegaard splittings it supports. Tao Li took a major step in that direction by showing that a non Haken 3---manifold has only finitely many irreducible Heegaard splittings. We will discuss an effective proof of this theorem and offer some thoughts on how it might be used to solve the classification problem. (Joint work with Toby Colding.)

8. **Karsten Grove, University of Notre Dame**
Title: Bruhat – Tits Geometry and Nonegative Curvature
Abstract: (Joint work with Fuquan Fang and Gudlaugur Thorbergsson.) There is a well-known link between (maximal) polar representations and isotropy representations of symmetric spaces provided by Dadok. Moreover, the theory by Tits and Burns-Spatzier provides a link between irreducible symmetric spaces of non-compact type of rank at least three and irreducible topological spherical buildings of rank at least three.
We discover and exploit a rich structure of a (connected) chamber system of finite (Coxeter) type M associated with any polar action of cohomogeneity at least two on any simply connected closed positively curved manifold. Although this chamber system is typically not a Tits geometry of type M, we prove that in all cases but one that its universal Tits cover indeed is a building. We construct a topology on this universal cover making it into a topological building in the sense of Burns and Spatzier. Using this structure we classify all polar actions on (simply connected) positively curved manifolds of cohomegeneity at least two.
For the much broader class of polar actions in nonnegative curvature, the structure of chamber systems plays a significant role as well for two of its three basic building blocks referred to as “affine”-, “spherical”- and “book”- polar. The above mentioned type of links for the “spherical” case have not been yet been established for the “affine case”, but a conjectural picture is emerging, indication that symmetric spaces are the source of all example of the “affine” kind as well. The “book” types are completely different and structurally completely understood.

9. **Ernst Heintze, University of Augsburg , Germany**
Title: Affine Kac-Moody algebras and symmetric spaces
Abstract: It is conjectured that also for affine Kac-Mooody algebras there is a theory of symmetric spaces, analogous to that in finite dimensions. We review some of the known results and describe in particular a classification of these spaces by pairs of involutions of compact simple Lie algebras.

10. **Hugo Parlier, University of Fribourg**
Title: The geometry of flip graphs of surface triangulations
Abstract: By taking an edge of a triangulation of a surface and flipping it, one gets a new triangulation of the surface. One way of measuring distance between

triangulations of the surface, with a prescribed set of vertices, is by counting the minimal number of ``flips" one must make to get from one to the other. A flip graph is the underlying graph with triangulations playing the part of vertices and edges are between triangulations that differ by a single flip. For instance, when the surface is a polygon, this graph is well-known combinatorial object, studied by many authors including Sleator, Tarjan and Thurston who proved precise results about its diameter more than 25 years ago. In general however the graph is infinite and is quasi-isometric to the underlying mapping class group of the surface (the group of self-homeomorphisms up to isotopy).

The talk is about the geometry of these graphs and their quotients by the associated mapping class groups. It will be about aspects of joint projects with Valentina Disarlo and Lionel Pournin.

11. **Sang-Hyun Kim, Seoul National University**
Title: Raags in Braids
Abstract: We show that every rightangled Artin group (RAAG) embeds into a RAAG defined by the opposite graph of a tree. We then embeds, by quasi isometries, an arbitrary RAAG into a pure braid group and also into the areapreserving diffeomorphism groups of the disk and of the sphere. Department of Mathematical Sciences, Seoul National University, Seoul, 151-747, Republic of Korea.

12. **John Loftin, Rutgers University**
Title: Cubic Differentials, Differential Geometry of Surfaces, and Representations of Surface Groups into Real Forms of *SL* (3，C)
Abstract: Given a cubic differential U on a Riemann surface with background conformal metric μ, perturb the metric to be e^u μ, where u satisfies the equation

$$\Delta_\mu u \pm 16||U||_\mu^2 e^{-2u} \pm 2e^u - 2_{K_\mu} = 0.$$

Then $e^u\mu$ is a natural Riemannian metric on a surface (either an affine sphere or a minimal Lagrangian surface in a Hermitian symmetric space, depending on the signs), and the structure equations of the surface can be integrated once the metric $e^u\mu$ and cubic differential U are in place. The structure equations give rise to a at connection on a principal bundle with structure group a real form G of $SL(3, \mathbb{C})$ and thus to a representation of the fundamental group into G. I will discuss the cases of a hyperbolic affine sphere $(signs+,-)$ and minimal Lagrangian surface in $\mathbb{CH}^2(signs-,-)$ in more detail, and describe the geometry of the surfaces and the corresponding representations (into $SL(3, \mathbb{R})$ and $SU(2,1)$ respectively). This study is naturally part of the relatively new field of Higher Teichmuller Theory, which seeks to extend the rich structure of Teichmuller space — and in particular the moduli of Fuchsian groups as representations of the fundamental group into $PSL(2,\mathbb{R})$ —to the moduli spaces of representations of surface groups into higher Lie groups.

13. **Zhi Lü, Fudan University**

Title: Equivariant Unitary Bordism and Equivariant Cohomology Chern Numbers

Abstract: (Joint work with Wei Wang.) By using the universal toric genus and the Kronecker pairing of bordism and cobordism, this paper shows that the integral equivariant cohomology Chern numbers completely determine the equivariant geometric unitary bordism classes of closed unitary G-manifolds, which gives an affirmative answer to the conjecture posed by Guillemin-Ginzburg-Karshon in [1, Remark H.5, ξ3, Appendix H], where *G* is a torus. Our approach heavily exploits Quillen's geometric interpretation of homotopic unitary cobordism theory. As a further application, we also obtain a satisfactory solution of [1, Question (A), ξ1.1, Appendix H] on unitary Hamiltonian G-manifolds. In particular, our approach can also be applied to the study of $(Z_2)^k$–equivariant unoriented bordism, and without the use of Boardman map, it can still work out the classical result of tom Dieck [2], which states that the $(Z_2)^k$–equivariant unoriented bordism class of a smooth closed $(Z_2)^k$-manifold is determined by its $(Z_2)^k$- equivariant Stiefel-Whitney numbers.

In addition, this paper also shows the equivalence of integral equivariant cohomology Chern numbers and equivariant K-theoretic Chern numbers for determining the equivariant unitary bordism classes of closed unitary G-manifolds by using the developed equivariant Riemann- Roch relation of Atiyah- Hirzebruch type, which implies that, in a different way, we may induce another classical result of tom Dieck [3], saying that equivariant K-theoretic Chern numbers completely determine the equivariant geometric unitary bordism classes of closed unitary G-manifolds.

References:

[1] V. Guillemin, V. Ginzburg and Y. Karshon, *Moment maps, cobordisms, and Hamiltonian group actions.* Appendix J by Maxim Braverman. Mathematical Surveys and Monographs, 98. American Mathematical Society, Providence, RI, 2002.

[2] T. tom Dieck, *Characteristic numbers of G-manifolds. I.* Invent. Math. 13 (1971), 213-224.

[3] T. tom Dieck, *Characteristic numbers of G-manifolds. II.* J. Pure Appl. Algebra 4 (1974), 31-39.

14. **Feng Luo, Rutgers University, USA**

Title: A dilogarithm identity on the moduli space of curves

Abstract: We establish an identity for closed hyperbolic surfaces whose terms depend on the dilogarithms of the lengths of simple closed geodesics in all 3-holed spheres and 1-holed tori in the surface. This is a joint work with Ser Peow Tan.

15. **Hideki Miyachi, Osaka University**
Title: Geometry of the Gromov Product in Teichm" Uller Theory
Abstract: In this talk, I will discuss the Thurston theory in extremal length geometry on Teichmüller space. Indeed, I will provide an explict connection between the Gromov product of the Teichmüller distance and the intersection number on the space of measured foliations. We also consider roughenings (or coarsenings) of isometries on Teichmüller space respecting the Gromov product and prove a rigidity property of such discretizations. As an application, we will give an alternative proof of a characterization of the isometry group on Teichmüller space, and a non-Minkowskian characterization (as Finsler space) of Teichmüller space.

16. **Ludovic Marquis , University of Rennes, France**
Title: Dehn filling of some finite volume hyperbolic 4-polytope
Abstract: A Coxeter d-polytope is the data P of a d-polytope of the real projective space RP d of dimension d and of a reflection σ s across each facet s of P with the extra condition that if the intersection s∩t of the facets s,t is of codimension 2 then the product σ s σ t is a rotation of finite order. Spherical, euclidean or hyperbolic polytopes with dihedral angles submultiple of π are the first examples of Coxeter polytopes. A Theorem of TitsVinberg shows that the group Γ generated by the reflections across the facets of a Coxeter polytope P is a discrete subgroup of PGL d+1 (R) preserving a convex open set **Ω** of RP d and P ∩ **Ω** is a fundamental domain for the action Γ y **Ω**. We exhibit examples of hyperbolic 4-polytope of finite volume which can be Dehn filled by Coxeter 4-polytope. More concretely, we exhibit a Coxeter group W, a converging sequence of discrete representations (ρ n) n of W such that: the representations (ρ n) n and ρ ∞ preserve a properly convex open set, the representations ρ n are not faithful but theirs images are Coxeter groups that act cocompactly on **Ω** n , the representation ρ ∞ is discrete and faithful, **Ω** ∞ is the hyperbolic space and the action on it is of finite covolume. We recall that Dehn filling of finite volume hyperbolic orbifold by compact hyperbolic orbifold are possible only in dimension 2 and 3. So, this work shows that in some peculiar examples one can Dehn filled hyperbolic orbifold if one is ready to go in the projective space. This is a joint work with Suhyoung Choi (KAIST) and GyeSeon Lee (Heidelberg).

17. **Kenichi Ohshika, Osaka University**
Title: Hyperbolic 3-manifolds and the Schottky space viewed as lying in the character variety
Abstract: For a fixed integer g ⩾ 2, we consider the character variety X g of repre-sentations of the free group of rank g into PSL 2 C. Both the Schottkybspace of rank g and the hyperbolic 3-manifolds whose fundamental

groups are generated by g elements are regarded as lying in X g . We shall study the behaviour of points corresponding to hyperbolic manifolds in X g . For instance the questions like how they can approach the Schottky space, how they can diverge, how they can accumulate into a space different from the Schottky space will be addressed. We shall give answers to these questions expressed in terms of the asymptotic behaviour of Heegaard splittings.

18. **Pedro Ontaneda, Binghamton University**
Title: Riemannian Hyperbolization
Abstract: The strict hyperbolization process of *R*. Charney and *M*. Davis produces a large and rich class of negatively curved spaces (in the geodesic sense). This process is based on an earlier version introduced by *M*. Gromov and later studied by *M*. Davis and *T*. Januszkiewicz. If *M* is a manifold its Charney-Davis strict hyperbolization is also a manifold, but the negatively curved metric obtained is far from being Riemannian because it has a large and complicated set of singularities. We show that these singularities can be removed (provided the hyperolization piece is large). Hence the strict hyperbolization process can be done in the Riemannian setting.

19. **Taras Panov , Lomonosov Moscow State University**
Title: Complex Geometry of Moment-angle Manifolds
Abstract: Moment-angle complexes are spaces acted on by a torus and parametrised by finite simplicial complexes. They are central objects in toric topology, and currently are gaining much interest in homotopy theory. Due the their combinatorial origins, moment-angle complexes also find applications in combinatorial geometry and commutative algebra. After an introductory part describing the general properties of moment-angle manifolds and complexes we shall concentrate on the complex-analytic aspects of the theory. Moment-angle manifolds provide a wide class of examples of non-K¨ ahler compact complex manifolds with interesting and complicated topology. A complex momentangle manifold Z is constructed via a certain combinatorial data, called a complete simplicial fan. In the case of rational fans, the manifold Z is the total space of a holomorphic bundle over a toric variety with fibres compact complex tori. By studying the Borel spectral sequence of this holomorphic bundle, we calculate the Dolbeault cohomology and Hodge numbers of Z. In general, a complex moment-angle manifold Z is equipped with a canonical holomorphic foliation F and an algebraic torus action transitive in the transverse direction. Examples of moment-angle manifolds include the Hopf manifolds, Calabi–Eckmann manifolds, and their deformations. We construct transversely K¨ ahler metrics on moment-angle manifolds, under some restriction on the combinatorial data. We prove that all K¨ ahler submanifolds in such a moment-angle manifold lie in a compact complex torus contained in a fibre of the foliation F. For a generic moment-angle manifold in its combinatorial class, we prove that all its subvarieties are moment-angle manifolds of smaller dimension. This implies, in

particular, that there are no non-constant meromorphic functions on such a manifold Z. This is joint work with Yuri Ustinovsky and Misha Verbitsky.

20. **Xiaochun Rong, Rutgers University & Capital Normal University**
Title: Equivariant Romov-hausdorff Convergence and Bounding Gromov Short Generators
Abstract: This is a preliminary report on a undergoing research project.

21. **Paul E. Schupp, University of Illinois**
Title: Computability, Complexity and Group Theory
Abstract: The remarkable interaction between group theory and questions of computability and computational complexity began with the famous paper of Max Dehn in 1912. Currently, the asymptotic-generic point of view of geometric group theory is shaping new ideas of computability and complexity. The concepts of being computable or computable in polynomial time are worst-case measures: How difficult is the hardest instance of the problem? The famous example showing that considering the worst-case does not give a good overall picture of a particular problem is Dantzig's Simplex Algo-rithm for linear programming which is used thousands of times every day and always runs very quickly. There are examples forcing the algorithm to take exponential time but one never sees such examples in practice. The introduction of generic-case complexity in 2003 showed that the classic word and conjugacy problems of group theory exhibit exactly this behavior- hard instances are very rare. I will discuss the development and examples of this idea and give an example of a new development in computability theory

22. **Weisu Su, Fudan University**
Title: the Horfunction Compactification of Teichm" Uller Spaces of Surfaces with Boundary
Abstract: The arc metric is an asymmetric metric on the Teichmüller space of a surface with nonempty boundary. It is the analogue of Thurston's metric on the Teichmüller space of a surface without boundary. We study the relation between Thurston's compactification and the horofunction compactification of Teichmüller space endowed with the arc metric. We prove that there is a natural homeomorphism between the two compactifications. This generalizes a result of Walsh for Thurston's metric on Teichmüller spaces of surfaces without boundary. The work is joint with D. ALESSANDRINI, L. LIU and A. PAPADOPOULOS.

23. **Ser Peow Tan, National University of Singapore**
Title: Polynomial Automorphisms of Cn Preserving the Markoff-hurwitz Polynomial
Abstract: We will talk about the action of the group of polynomial auto-morphisms of C^n ($n \geqslant 3$) which preserve the Markoff-Hurwitz polynomial $H(x) := x_1^2 + \cdots + x_n^2 - x_1 x_2 \ldots x_n$ .We will discuss the

determination of the group and its action on C n ; the description of a non-empty open subset of C n on which the group acts properly discontinuously (domain of discontinuity); and identities for the orbit of points in the domain of discontinuity. This is joint work with Hengnan Hu and Ying Zhang.

24. **Nikolai Vavilov, Saint Petersburg State University**
Title: Decomposition of unipotents, revisited
Abstract: The talk is devoted to the recent versions and new applications of decomposition of unipotents. Let Φ be a root system, R be a commutative ring, and let G(Φ,R) be a Chevalley group of type Φ over R. Further, denote by $x_\alpha(\xi)$, where $\alpha \in \Phi$ and $\xi \in R$, the corresponding elementary root element.
For the group SL(n,R) this method was initially proposed in 1987 by Alexei Stepanov, to give a simplified proof of Suslin's normality theorem. Soon there-after I generalised it to split classical groups, and then together with Eugene Plotkin we generalised it to exceptional Chevalley groups.
Technically, decomposition of unipotents relies on the possibility to find enough small unipotents in the stabilisers of vectors in certain orbits of Chevalley groups in small representations, such as the microweight ot the adjoint ones.
In the simplest form, this method gives explicit polynomial factorisations of root type elements $gx_\alpha(\xi)g^{-1}$, where $g \in G(\Phi,R)$, in terms of factors sitting in proper parabolic subgroups, and eventually in terms of elementary generators. Among other things, this allows to give extremely short and straightforward proofs of the main structure theorems for such groups.
However, for exceptional groups the early versions of the method relied on the presence of huge classical embeddings, such as $A_5 \leqslant E_6$, $A_7 \leqslant E_7$ and $D_8 \leqslant E_8$. Also, even for some classical groups, the method would not give explicit bounds on the length of elementary decompositions of root elements. Inspired by the A_2-proof of structure theorems for Chevalley groups, proposed by myself, Mikhail Gavrilovich, Sergei Nikolenko, and Alexander Luzgarev, recently we succeeded in developing new powerful versions of the method, that only depend on the presence of small rank classical embeddings. I plan to describe some of these recent advances.

- A_3-proof, D_4 -proof, etc. for Chevalley groups of types E_6 and E_7.
- Explicit linear length factorisation of transvections for Sp(2l,R).
- Applications to description of overgroups of subsystem subgroups, in particular, for the case $A_7 \leqslant E_7$ (joint with Alexander Shchegolev).
- Description of subnormal subgroups of Chevalley groups of types E_6 and E_7 (joint with Zuhong Zhang).

Finally, I plan to discuss possible generalisations to isotropic (but non necessarily split) reductive groups. In this generality, Victor Petrov and Anastasia Stavrova initiated the use of localisation methods, further developed by Stavrova, Luzgarev, Stepanov, and others. Decomposition of unipotents would constitute an viable alternative to and in some cases enhancement of localisation methods, and might have serious impact on the structure theory of such groups. Actually, it many important cases it gives much better length bounds in terms of elementary generators. Another tantalising challenge would be to extend these methods to infinite-dimensional algebraic-like groups such as Kac—Moody groups, etc.

25. **Yuefei Wang, Institute of Mathematics, AMSS, Chinese Academy of Sciences**
Title: Semigroups of Holomorphic Maps on Berkovich Projective Space
Abstract: In recent years there is a considerable interest on the semigroup actions on non-Archimedean fields, or p-adic dynamics. Nevertheles the non-Archimedean fields have rather poor topological properties, which are totally disconnected and not locally compact. In 1990 Berkovich introduced a new and rather nice topological space, which is compact, path-connected and contains the non-Archimedean fields as dense subspaces in the sense of Gel'fand topology. Berkovich's general approach to non-Archimedean analytic geometry has various important applications to geometry, number theory and dynamics, etc. In this talk we will first give an introduction to the Berkovich projective space, and then discuss the dynamics over the space, in comparison with the complex holomorphic dynamics. We will also talk about problems and recent results on p-adic Julia and Fatou sets, and the minimality and minimal decomposition of p-adic rational dynamical systems.

26. **Michael Wolf, Rice University**
Title: Polynomial Pick forms for affine spheres, real projective polygons and asymptotics in Hitchin components
Abstract: (Joint work with David Dumas.) Discrete surface group representations into PSL (3, R) correspond geometrically to convex real projective on surfaces; in turn, these may be studied by considering the affine spheres which project to the convex hull of their universal covers. As a sequence of convex projective structures leaves all compacta in its deformation space (the Hitchin component for PSL (3, R), a subclass of the limits is described by polynomial cubic differentials on affine spheres which are conformally the complex plane. We show that those particular affine spheres project to polygons; along the way, a strong estimate on asymptotics is found, which translates to a version of the Stokes data. We begin by describing the basic objects and context and conclude with a sketch of both some of the useful technique and some of its recent applications to other settings.

27. **Yunfei Wu, Rice University**
Title: Translation lengths of parabolic isometries of CAT(0) spaces and its application to the geometry and topology of Hadamard manifolds
Abstract: A CAT(0) space is a complete path-metric space with a certain inequality property. In this talk, we will discuss the translation length of parabolic isometries of CAT(0) spaces. As an application, we will connect several open problems and conjectures on Hadamard manifolds and moduli spaces of curves.