# Random walks arising in statistical physics

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**Abstract:**

**Part 1:Random walks arising in statistical physics**

I have been studying random paths with strong interactions arising in statistical physics for over forty years. I will survey of the program and how much we have learned in this time. I will discuss the idea of scaling limits (which leads, say, to the construction of the Schramm-Loewner evolution), the interplay between the discrete and the continuous, and the relationship between the spatial dimension of the ambient space and the fractal dimension of the paths. The talk will also discuss questions that we do not know how to answer and are topics for future research.

**Part 2: Loop measures, Gaussian fields and determinants**

Much of the analysis of random paths especially in two dimensions has considered the underlying "field" that generate the paths. Two closely related basic fields are the Gaussian free field and loop measures (and soups). I will give an introduction to these in both discrete and continuous settings and give the relationship with random paths (SLE) and the determinant of the Laplacian.

**Part 3: Fractal measures**

I will discuss the construction and analysis of fractal measures such as the occupation time or natural parametrization (Minkowski content) of the Schramm-Loewner evolution (SLE) and exceptional points of Brownian motion as well as measures arising from exponentiating Gaussian fields (Liouville quantum gravity).

## Massless phases for the Villain model in $d \geq 3$

吴炜 (We Wei)
NYU Shanghai

**Abstract:** The XY and the Villain models are mathematical idealization of real world models of liquid crystal, liquid helium, and superconductors. Their phase transition has important applications in condensed matter physics and led to the Nobel Prize in Physics in 2016. However we are still far from a complete mathematical understanding of the transition. The spin wave conjecture, originally proposed by Dyson and by Mermin and Wagner, predicts that at low temperature, large scale behaviors of these models are closely related to Gaussian free fields. I will review the historical background and discuss some recent progress on this conjecture in $d \geq 3$. Based on the joint work with Paul Dario (Tel Aviv).

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## Large $N$ Limit of the $O(N)$ Linear Sigma Model via Stochastic Quantization

朱蓉婵 (Zhu Rongchan)

Beijing Institute of Technology

**Abstract:** In this talk we discuss large $N$ limits of a coupled system of $N$ interacting $\Phi^4$ equations posed over $\mathbb{T}^d$ for $d = 1,2,3$, known as the $O(N)$ linear sigma model. Uniform in $N$ bounds on the dynamics are established, allowing us to show convergence to a mean-field singular SPDE, also proved to be globally well-posed. Moreover, we show tightness of the invariant measures in the large $N$ limit. For large enough mass, they converge to the (massive) Gaussian free field, the unique invariant measure of the mean-field dynamics, at a rate of order $1/\sqrt{N}$ with respect to the Wasserstein distance. We also consider fluctuations and obtain tightness results for certain $O(N)$ invariant observables, along with an exact description of the limiting correlations in $d = 1,2$. This talk is based on joint work with Hao Shen, Scott Smith and Xiangchan Zhu.

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## Fluctuations on Plancherel integer partitions around its limit shape

苏中根 (Su Zhonggen)

Zhejiang University

**Abstract:** For a natural number $n$, let $\mathscr{P}_n$ be the space of all integer partitions $\lambda$ of $n$, namely $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)$ such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda$ and $\lambda_1 + \lambda_2 + \cdots + \lambda_l = n$. Let $P_{pl}(\lambda) = \frac{d_\lambda^2}{n!}$, where $d_\lambda$ stands for the numbers of all standard Young tableaux with shape $\lambda$. A remarkable result, almost simultaneously obtained by Logan and Shepp, Vershik and Kerov in the seventies, is that there is a limit shape $\omega(x)$ for suitably scaled $\lambda$ under the probability measure $P_{pl}$. In this talk we will report a Gaussian fluctuation result for $\lambda_{[\sqrt{n}x]}$ around the shape curve $\omega(x)$. The result complements, in a striking way, the well-known theorem of Kerov on the generalized Gaussian convergence. The proofs are based on the poissonization techniques and the Costin–Lebowitz–Soshnikov central limit theorem for determinantal point processes.

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# Limit set of branching random walks on hyperbolic groups

向开南 (Xiang Kainan)

Xiangtan University

**Abstract:** Let $\Gamma$ be a nonelementary hyperbolic group with a word metric $d$ and $\partial\Gamma$ its hyperbolic boundary equipped with a visual metric $d_a$ for some parameter $a > 1$. Fix a superexponential symmetric probability $\mu$ on $\Gamma$ whose support generates $\Gamma$, and denote by $\rho$ the spectral radius of the random walk $Y$ on $\Gamma$ with step distribution $\mu$. Let $\nu$ be a probability on $\{1, 2, 3, \ldots\}$ with mean $\lambda = \sum_{k=1}^{\infty} k\nu(k) < \infty$.

Let BRW$(\Gamma, \nu, \mu)$ be the branching random walk on $\Gamma$ with offspring distribution $\nu$ and base motion $Y$. Write $\Lambda$ for the limit random subset of $(\partial\Gamma, d_a)$ consisting of all accumulation points of the trace of BRW$(\Gamma, \nu, \mu)$. Denote by $H$ the volume growth rate for the trace of BRW$(\Gamma, \nu, \mu)$ and by $h$ the Hausdorff dimension of the limit set $(\Lambda, d_a)$. We prove the following two universality results:

**(a)** $H$ is almost surely a deterministic, strictly increasing and continuous function of $\lambda \in [1, 1/\rho]$, is bounded by the square root of the volume growth rate of $\Gamma$, and has critical exponent $1/2$ at $1/\rho$ in the sense that for some positive constant $C$,

$$H(1/\rho) - H(\lambda) \sim C\sqrt{1/\rho - \lambda} \quad \text{as } \lambda \uparrow 1/\rho.$$

**(b)** For BRW$(\Gamma, \nu, \mu)$ with $\lambda \in [1, 1/\rho)$, almost surely $h = \log_a H(\lambda)$. Thus $h(\cdot)$ has critical exponent $1/2$ near $1/\rho$ in a "weak" sense that

$$h(1/\rho-) - h(\lambda) \sim \frac{C}{H(1/\rho)\log a}\sqrt{1/\rho - \lambda} \text{ as } \lambda \uparrow 1/\rho.$$

We conjecture that the Hausdorff dimension of $(\Lambda, d_a)$ in the critical case $\lambda = 1/\rho$ is $\log_a H(1/\rho)$ almost surely. This has been confirmed on free groups or the free product (by amalgamation) of finitely many finite groups.

This talk is based on a joint work with Sidoravicius Vladas and Wang Longmin.

## Asymptotics for critical and near-critical first-passage percolation on the triangular lattice

姚昌龙 (Yao Changlong)
Academy of Mathematics and Systems Science

**Abstract:** We study Bernoulli first-passage percolation on the triangular lattice in which sites have 0 and 1 passage times with probability $p$ and $1-p$, respectively. We consider the following three cases: (1) $p = p_c = 1/2$. Let $a(0,n)$ denote the first-passage time from the origin to the point $n$. We obtain explicit limit theorems for $a(0,n)/\log(n)$ and $\mathrm{Var}[a(0,n)]/\log(n)$. The proof relies on the conformal loop ensemble CLE(6) and a color switching trick. (2) $p > p_c$. We give exact asymptotics for the first-passage times from the origin to the infinite cluster with 0-time sites, as $p$ tends to $p_c$ from above. The proof uses the result in the critical case and Russo's formula. (3) $p < p_c$. Let $L(p)$ denote the correlation length, and let $B(p)$ denote the limit shape in the classical shape theorem. We show that the rescaled limit shape $B(p)/L(p)$ converges to a Euclidean disk, as p tends to $p_c$ from below. The proof relies on the scaling limit of near-critical percolation established by Garban, Pete and Schramm (2018) and the construction of the collection of continuum clusters introduced by Camia, Conijn and Kiss (2019).

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## On the chemical distance exponent for the two-sided level-set of the 2D Gaussian free field

高一帆 (Gao Yifan)
Peking University

**Abstract:** We introduce the chemical distance for the two-sided level-set percolation of the two-dimensional discrete Gaussian free field on a box $V_N$ of side length $N$. We will show that for some $\varepsilon > 0$, with probability tending to 1 as $N$ goes to infinity, there exists no open path of length less than $N^{1+\varepsilon}$ connecting two vertices with macroscopic distance. This talk is based on a joint work with Fuxi Zhang.

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## Crossing probability in Gaussian Free field

刘明昶 (Liu Mingchang)
Tsinghua university

**Abstract:** Two-dimensional Gaussian free field (GFF) is a natural 2D time analogue of Brownian motion. Crossing probability is a delicate quantity for GFF. We will introduce discrete GFF(dGFF) and metric graph GFF(mGFF). Both objects converge to continuum GFF as distributions. However, the crossing probabilities in dGFF and in mGFF are distinct. In this talk, we focus on the mGFF case. It turns out that the crossing probability in mGFF converges to the "fusion" of the so-called pure partition functions. Moreover, we find that the scaling limits of crossing probabilities satisfy a third-order PDE system.
This talk is based on a joint work with Hao Wu.

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## Cluster explorations of the loop soup on a metric graph related to the Gaussian free field

Elie Aidekon
NYU Shanghai

**Abstract:** We give a Markov property for the loop soup on a metric graph which mimics that of the Gaussian free field, and describe the law of the loop soup conditionally on its occupation field.

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## Asymptotics of the Pearcey determinant

张仑 (Zhang Lun)
Fudan University

**Abstract:** The Pearcey kernel is a classical and universal kernel arising from random matrix theory, which describes the local statistics of eigenvalues when the limiting mean eigenvalue density exhibits a cusp-like singularity. It appears in a variety of statistical physics models beyond matrix models as well. In this talk, we are concerned with the Fredholm determinant $\det\left(I-\gamma K_{s,\rho}^{\mathrm{Pe}}\right)$, where $0\leq\gamma\leq 1$ and $K_{s,\rho}^{\mathrm{Pe}}$ stands for the trace class operator acting on $L^2(-s,s)$ with the Pearcey kernel. We obtain asymptotics of this determinant as $s\to+\infty$, which is also interpreted as large gap asymptotics in the context of random matrix theory. It comes out that the Pearcey determinant exhibits a significantly different asymptotic behavior for $\gamma=1$ and $0<\gamma<1$, which suggests a transition will occur as the parameter $\gamma$ varies. This talk is based on two recent joint works with Dan Dai and Shuai-Xia Xu.

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