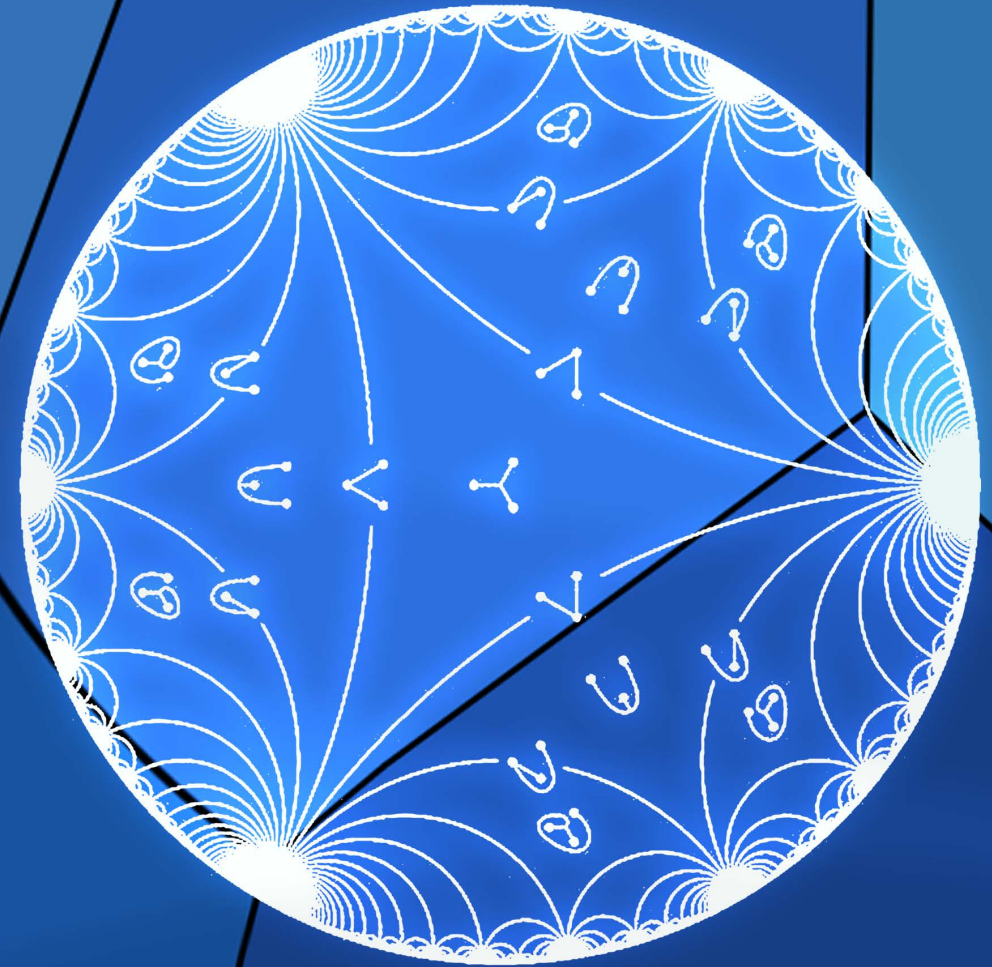


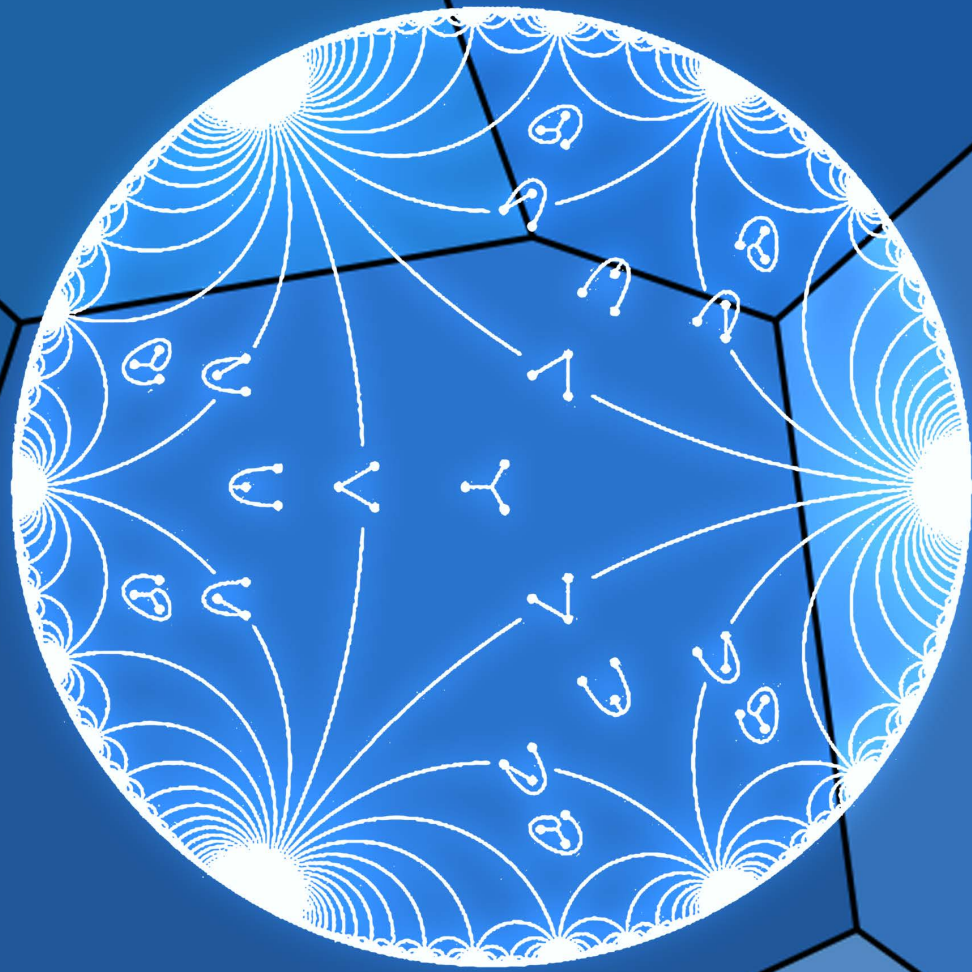


# 复动力系统与复方程研讨会

## Workshop on Complex Dynamics & Complex Equations

2023.07.22 - 29





## Schedule for Workshop on Complex Dynamics & Complex Equations

July 22–29, 2023

Time & Date	周日 (7.23)	周一 (7.24)	周二 (7.25)	周三 (7.26)	周四 (7.27)	周五 (7.28)
7:30–8:30	Breakfast (60 minutes)					
8:30–10:00	Jianhua Zheng 郑建华	Fei Yang 杨飞	Jianhua Zheng 郑建华	Free Discussion	Tingbin Cao 曹廷彬	Chengfa Wu 吴成发
10:00–10:30	Coffee Break (within 30 minutes)				Coffee Break (within 30 minutes)	
10:30–12:00	Jianhua Zheng 郑建华	Fei Yang 杨飞	Jianhua Zheng 郑建华		Tingbin Cao 曹廷彬	Chengfa Wu 吴成发
12:00–14:00	Lunch (120 minutes)					
14:00–15:00	Zhibo Huang 黄志波	Zhaojun Wu 吴昭君	Fei Yang 杨飞	Free Discussion	Chengfa Wu 吴成发	Xiaomin Li 李效敏
15:00–15:30	Coffee Break (within 30 minutes)				Coffee Break (within 30 minutes)	
15:30–16:30	Jiaxing Huang 黄家兴	Zhitao Wen 温智涛	Fei Yang 杨飞		Chengfa Wu 吴成发	Guowei Zhang 张国威
16:30–17:30	Shuangting Lan 蓝双婷	Free Discu- ssion		(闭幕)		
17:30	Banquet 18:00–20:00	Dinner				

## 1 郑建华 (清华大学)

### 题目: 亚纯函数的游荡域

**摘要:** 近两年国际上对亚纯函数的游荡域有深度的研究, 我们来介绍他们的工作, 将从游荡域的基本概念出发展开介绍, 主要在两个方面: 一是游荡域的分类; 二是特殊游荡域的构造。游荡域的分类是借助双曲度量来实施的, 共分为九类, 并用逼近论的方法构造出每类游荡域。我们主要是学习他们的方法, 考虑将这些方法应用于其他问题的研究上, 所以我们注重方法的介绍和评注。

## 2 杨飞 (南京大学)

**Topic:** Local connectivity of Julia sets of some rational maps with Siegel disks

**Abstract:** In a joint work with Shuyi Wang, Gaofer Zhang and Yanhua Zhang, we prove that a long iteration of rational maps is expanding near boundaries of bounded type Siegel disks. This leads us to extend Petersen's local connectivity result on the Julia sets of quadratic Siegel polynomials to a general case. A new key feature in the proof is that the puzzles are not used.

## 3 温智涛 (汕头大学)

**Topic:** Finite order solutions to linear differential equations with exponential sum coefficients

**Abstract:** We say that  $f$  is of completely regular growth if there exists a sequence of Euclidean discs  $D(z_k, r_k)$  satisfying

$$\sum_{|z_k| \leq r} r_k = o(r)$$

such that

$$\log |f(re^{i\theta})| = (h_f(\theta) + o(1))r^{\rho(f)}, \quad re^{i\theta} \notin \bigcup_k D(z_k, r_k),$$

as  $r \rightarrow \infty$  uniformly in  $\theta$ . Gol'dberg and Ostrovskii stated the following problem [2, p. 300]:

**Gol'dberg-Ostrovskii's Problem.** Suppose that  $f$  is a finite order transcendental solution of

$$f^{(n)} + a_{n-1}(z)f^{(n-1)} + \cdots + a_1(z)f' + a_0(z)f = 0 \quad (1)$$

whose coefficients are entire functions of completely regular growth. Is it true that  $f$  is of completely regular growth?

Petrenko [3, pp. 104–112] has shown that transcendental solutions to linear differential equations (1) with polynomial coefficients are of completely regular growth. However, Bergweiler [1] constructs a counter-example to show Gol'dberg-Ostrovskii's Problem is not true.

In this talk, we show that if  $f$  is a finite order transcendental solution of (1) with exponential sum coefficients with rational leading coefficients, then  $f$  is of complex regular growth. Moreover, the representation of the transcendental entire solution is given, that is,  $f$  is also an exponential sum.

## References

- [1] Bergweiler, W., *A question of Goldberg and Ostrovskii concerning linear differential equations with coefficients of completely regular growth*. Proc. Amer. Math. Soc. **151** (2023), no. 5, 2097–2101.
- [2] Havin, V. P. and Nikolski, N. K., *Linear and Complex Analysis. Problem Book 3. Part II*. Lecture Notes in Mathematics, 1574. Springer-Verlag, Berlin, 1994.
- [3] Petrenko, V. P., *Entire Curves*. Vishcha Shkola, Kharkov, 1984. (Russian)

#### 4 吴成发 (深圳大学)

##### 授课大纲:

##### Lecture 1: An introduction to elliptic functions and their applications in complex differential equations

In the first part of this lecture, we will briefly introduce the theory of elliptic functions.

The second part of this lecture is devoted to applications of elliptic functions in complex differential equations. First, we discuss classifications of meromorphic solutions of certain autonomous complex differential equations. In particular, we will focus on the Loewy factorizable algebraic ODEs. Then we move to the study of the autonomous Schwarzian differential equations (SDEs). Ishizaki showed that there are six canonical types of autonomous SDEs that have transcendental meromorphic solutions. We will construct all their transcendental meromorphic solutions explicitly. In particular, the solutions of four types are shown to be elliptic functions.

##### Lecture 2: Rational solutions of the Painlevé equations and their associated polynomials

This lecture will review rational solutions of the Painlevé equations and their associated special polynomials. In particular, the root structures of these special polynomials will be discussed. If time permits, we will also discuss rogue wave patterns, which are closely related to the root structures of certain special polynomials.

#### 5 蓝双婷 (广东技术师范大学)

##### Topic: Fixed points and Picard values of meromorphic functions and their differences

**Abstract:** Fixed point theory is an important object of study in function theory. Under some conditions, we obtained the relationships between the exponent of convergence of fixed points of meromorphic function, its shift and forward differences, which can be regarded as difference counterpart of one of Xinhua's result. Picard theorem is one of important results in module

distribution. MmClunie gave the exact form of transcendental meromorphic function, when and its derivatives have a finite number of zeros and poles. We consider the difference counterpart of Clunie's result, and give the exact forms of transcendental entire function and its forward difference, when both and have finitely many zeros without growth condition.

#### 6 张国威 (安阳师范学院)

##### Topic: On Petrenko's deviations and the Julia limiting directions of solutions of complex differential equations

**Abstract:** The Julia set  $J(f)$  of a transcendental meromorphic function  $f$  has chaotic behavior in complex dynamics. The ray  $\arg z = \theta \in [0, 2\pi)$  is said to be a limiting direction of  $J(f)$  if there is an unbounded sequence  $\{z_n\} \subseteq J(f)$  such that  $\lim_{n \rightarrow \infty} \arg z_n = \theta$ . Petrenko's deviation are given by  $\beta_-(\infty, f) = \liminf_{r \rightarrow \infty} \log \frac{M(r, f)}{T(r, f)}$  and  $\beta_+(\infty, f) = \limsup_{r \rightarrow \infty} \log \frac{M(r, f)}{T(r, f)}$ . In this report, we mainly talk about the Julia limiting directions of entire solutions of complex differential equations, of which coefficient is associated with Petrenko's deviation. In fact, we obtained the lower bounds of the measure on the sets of Julia limiting directions of these solutions have closed relations with Petrenko's deviation of the coefficient.

## 7 李效敏 (中国海洋大学)

**Topic:** Meromorphic solutions of the differential equation  $(f^n)^{(k)}(g^n)^{(k)} = 1$  and its applications

**Abstract:** In 1997, C.C. Yang and X.H. Hua [Ann. Acad. Sci. Fenn. Math. **22**(1997), no.2, 395-406] proved that if  $f$  and  $g$  are two nonconstant meromorphic functions such that  $f^n f' g^n g' = 1$ , where  $n$  is a positive integer satisfying  $n \geq 6$ , then  $f$  and  $g$  are transcendental entire functions such that  $g(z) = c_1 e^{cz}$  and  $f(z) = c_2 e^{-cz}$ , where  $c, c_1$  and  $c_2$  are constants such that  $(c_1 c_2)^{n+1} c^2 = -1$ . By Zalcman's Lemma, we prove that if  $f$  and  $g$  are two nonconstant meromorphic functions such that  $(f^n)^{(k)}(g^n)^{(k)} = 1$ , where  $n$  and  $k$  are positive integers satisfying  $n > 2k$ , then  $f$  and  $g$  are transcendental entire functions such that  $f(z) = c_1 e^{cz}$  and  $g(z) = c_2 e^{-cz}$ , where  $c_1, c_2$  and  $c$  are nonzero constants satisfying  $(-1)^k (c_1 c_2)^n (nc)^{2k} = 1$ . Applying the result, we completely resolve a uniqueness question of meromorphic functions concerning certain nonlinear differential polynomials. As the applications of one of the main results in this paper, we also improve Theorem 1 from C.C. Yang and X.H. Hua [Ann. Acad. Sci. Fenn. Math. **22**(1997), no.2, 395-406] and study a periodicity question of nonconstant meromorphic functions concerning certain nonlinear differential polynomials, where the periodicity question is related to a Yang's conjecture introduced in Q. Wang and P. C. Hu [Acta Math. Sci. **38**(2018), no.2, 209-214] and the differential-difference versions of the Yang's conjecture proposed in X. L. Liu and R. J. Korhonen [Bull. Australian Math. Soc. **101**(2020), no.3, 453-465]. Our reasoning in this paper will make up the gap in the proof of Theorem 2 from S.S. Bhoosnurmath and R.S. Dyavanal [Comput. Math. Appl. **53**(2007), no. 8, 1191-1205].

## 8 吴昭君 (湖北科技学院)

**题目:** 一类复时滞微分多项式 (时滞微分方程解) 的值分布

**摘要:** 讨论了具有最大亏量和的亚纯函数的时滞微分多项式的值分布, 同时介绍一类线性时滞微分方程解去小函数的零点收敛指数。

## 9 黄志波 (华南师范大学)

**Topic:** Growth of solutions to higher order differential equations with regular growth coefficients

**Abstract:** The classical problem of finding conditions on the entire coefficients  $A_j$  ( $j = 0, 1, \dots, k-1$ ) ensuring that all nontrivial solutions to higher order differential equations  $f^{(k)} + A_{k-1} f^{(k-1)} + \dots + A_1(z) f' + A_0(z) f = 0$  are of infinite lower order or infinite order is being discussed in this paper. In particular, we assume that the coefficients (or most of them) are Mittag-Leffler functions or exponential polynomials, which are of regular growth.

## 10 曹廷彬 (南昌大学)

**题目:** 值分布理论的离散化与复时滞微分方程

**摘要:** 报告分为两个部分。第一部分主要介绍关于值分布理论的离散化的最新动态与进展情况; 第二部分主要介绍复时滞微分方程亚纯解理论的最新成果。

## 11 黄家兴 (深圳大学)

**Topic:** Oscillating simply connected wandering domains and periodic domains

**Abstract:** The geometry of simply connected wandering domains for entire functions has been studied by Luka Boc Thaler, who showed that every bounded connected regular open set, whose closure has a connected complement, is an oscillating or escaping wandering domain of some entire function. It is natural to ask if such a domain can be realized as a periodic domain. In this talk, we will answer this question. The results are joint work with Prof. Jian-Hua Zheng.

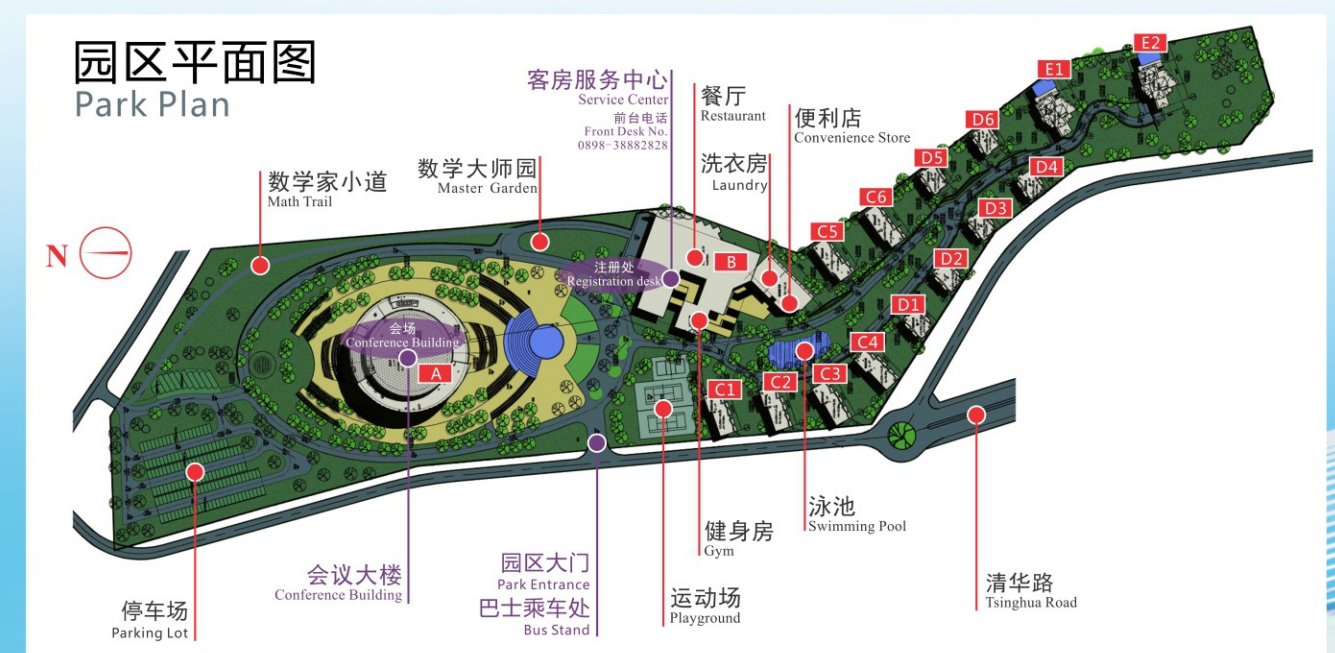
## Welcome to TSIMF



The facilities of TSIMF are built on a 23-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 29,000 square meter that includes state-of-the-art conference facilities (over 10,000 square meter) to hold many international workshops

simultaneously, two libraries, a guest house (over 10,000 square meter) and the associated catering facilities, a large swimming pool, workout gym and sport courts and other recreational facilities.

Mathematical Sciences Center (MSC) of Tsinghua University, assisted by TSIMF's International Advisory Committee and Scientific Committee, will take charge of the academic and administrative operation of TSIMF. The mission of TSIMF is to become a base for scientific innovations, and for nurturing of innovative human resource; through the interaction between leading mathematicians and core research groups in pure mathematics, applied mathematics, statistics, theoretical physics, applied physics, theoretical biology and other related fields, TSIMF will provide a platform for exploring new directions, developing new methods, nurturing mathematical talents, and working to raise the level of mathematical research in China.



## Restaurant

All the meals are provided in the restaurant (Building B1) according to the time schedule.



**Breakfast**  
07:30-08:30

**Lunch**  
12:00-13:30

**Dinner**  
17:30-19:00

## Laundry

Opening Hours: 24 hours  
The self-service laundry room is located in the Building 1 (B1).



## Gym

The gym is located in the Building 1 (B1), opposite to the reception hall. The gym provides various fitness equipment, as well as pool tables, tennis tables etc.

## Playground



Playground is located on the east of the central gate. There you can play basketball, tennis and badminton. Meanwhile, you can borrow table tennis, basketball, tennis balls and badminton at the reception desk.

## Swimming Pool



Please note that there are no lifeguards. We will not be responsible for any accidents or injuries. In case of any injury or any other emergency, please call the reception hall at +86-898-38882828.

## Outside Shuttle Service

We have shuttle bus to take participants to the airport for your departure service. Also, we would provide transportation at the Haihong Square (海虹广场) of Howard Johnson for the participants who will stay outside TSIMF. If you have any questions about transportation arrangement, please feel free to contact Ms. Li Ye (叶莉) at (0086)139-7679-8300.

## Free Shuttle Bus Service at TSIMF

We provide free shuttle bus for participants and you are always welcome to take our shuttle bus, all you need to do is wave your hands to stop the bus.



Destinations: Conference Building, Reception Room, Restaurant, Swimming Pool, Hotel etc.



## Contact Information of Administration Staff

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