

# Titles and Abstracts

Volker Branding, University of Wien

**Title:** Global existence of Dirac-wave maps with curvature term on expanding space-times

**Abstract:** The action functional of the supersymmetric nonlinear sigma model is an important model in modern quantum field theory. From a mathematical point of view it consists of a map between two manifolds and a vector spinor defined along that map.

If one chooses a Riemannian domain manifold its critical points couple the elliptic harmonic map equation with the vector spinor, this system became known as Dirac-harmonic maps and variants thereof. Since the action functional of the supersymmetric nonlinear sigma model is unbounded from below it is very difficult to obtain existence results for this system.

However, in the case of a globally hyperbolic domain manifold the critical points lead to the Dirac-wave map system which is a hyperbolic system of partial differential equations. In this setup unbounded action functionals are much better to deal with.

We will present an existence result for wave maps and Dirac-wave maps with curvature term with small initial data on globally hyperbolic manifolds of arbitrary dimension which satisfy a suitable growth condition. This is joint work with Klaus Kröncke.

Qun Chen, Wuhan University

**Title:** Existence for Dirac equations and Dirac-harmonic maps

**Abstract:** Dirac-harmonic maps are solutions of a coupled system of a Dirac equation and harmonic map type equation, the existence for which has been a challenging problem for years. In this talk, we will discuss the existence and uniqueness of solutions of Dirac equations satisfying a class of local elliptic boundary value conditions, and the existence of Dirac-harmonic maps and their heat flows. This is based on joint works with J.Jost, J.Y.Li, L.L.Sun, G.F.Wang and M.M.Zhu respectively.

Zhijie Chen, Tsinghua University

**Title:** On the curvature equation with singular sources on flat tori

**Abstract:** I will introduce some recent advances on the curvature equation

$$\Delta u + e^u = \rho \delta_0 \text{ on } E_\tau, \quad \rho > 0,$$

where  $E_\tau$  is a flat torus and  $\delta_0$  is the Dirac measure at the lattice point. The interesting point of this problem is that, when  $\rho$  takes a critical value, i.e.  $\rho \in 8\pi\mathbb{N}$ , then the solvability depends on the geometry of the torus (i.e. the moduli  $\tau$ ) in a sophisticated manner. This is based on joint works with Professor Chang-Shou Lin.

Dong Ye, Université de Lorraine

**Title:** TBA

**Abstract:** TBA

Yuxin Dong, Fudan University

**Title:** On Eells-Sampson type theorems for subelliptic harmonic maps

**Abstract:** A sub-Riemannian manifold is a manifold with a subbundle of the tangent bundle and a fiber metric on this subbundle. A Riemannian extension of a sub-Riemannian manifold is a Riemannian metric on the manifold compatible with the fiber metric on the subbundle. One may define an analog of the Dirichlet energy by replacing the L2 norm of the derivative of a map between two manifolds with the L2 norm of the restriction of the derivative to the subbundle when the domain is a sub-Riemannian manifold. A critical map for this energy is called a subelliptic harmonic map. In this talk, by use of a subelliptic heat flow, we establish some Eells-Sampson type existence results for subelliptic harmonic maps when the target Riemannian manifold has non-positive sectional curvature.

Huijun Fan, Peking University

**Title:** TBA

**Abstract:** TBA

Jixiang Fu, Fudan University

**Title:** Limiting behavior of a class of Hermitian Yang-Mills metrics

**Abstract:** This talk concerns Hermitian Yang-Mills connections on a class of rank 2 holomorphic vector bundles over the product  $X$  of two copies of the complex one torus  $T$  and  $B$ . The vector bundles are stables with respect to a family of Kähler metrics on  $X$  which are flat and have areas  $\epsilon$  and  $\epsilon^{-1}$  on  $T$  and  $B$ , respectively. We will study the asymptotic behavior of the resulting Hermitian Yang-Mills connections  $H_{1,\epsilon}$  by constructing a family of Hermitian metrics  $H_{0,\epsilon}$  and doing the  $C^k$ -estimate to the difference between  $H_{0,\epsilon}^{-1}H_{1,\epsilon}$  and the identity matrix. In this talk we will focus on the  $C^0$ -estimate.

Min-Chun Hong, University of Queensland

**Title:** Some results on the Ericksen-Leslie system for the Oseen-Frank model in liquid crystals.

**Abstract:** In this talk, I will discuss some results on the Ericksen-Leslie system for the Oseen-Frank model with unequal Frank elastic constants in  $\mathbb{R}^3$ . In particular, I discuss my joint work with Dr Yu Mei on the existence of solutions to the Ericksen-Leslie system with initial data having small local  $L^3$ -norm. Moreover, I will also discuss my recent result with Feng, and Mei about the convergence of the Ginzburg-Landau approximation system to the Ericksen-Leslie system.

Gerhard Huisken, University of Tbingen & MFO

**Title:** Inverse mean curvature flow of entire graphs

**Abstract:** The lecture describes joint work with P. Daskalopoulos on existence and longtime behavior of solutions to inverse mean curvature flow, that are given as entire graphs. In particular, we explain new a priori estimates using fast diffusion properties of the flow.

Lei Liu, Max Planck Institute for Mathematics in the Sciences

**Title:** Existence and asymptotic analysis for solutions of a mixed elliptic-parabolic boundary value problem coupling a harmonic-like map with a nonlinear spinor

**Abstract:** In this talk, we solve a new elliptic-parabolic system arising in geometric analysis that is motivated by the nonlinear supersymmetric sigma model of quantum field theory. The corresponding action functional involves two fields, a map from a Riemann surface into a Riemannian manifold and a spinor coupled to the map. The first fields has to satisfy a second order elliptic system, which we turn into a parabolic system so as to apply heat flow techniques. The spinor, however, satisfies a first order Dirac type equation. We carry that equation as a nonlinear constraint along the flow.

With this novel scheme, in more technical terms, we can show the existence of Dirac-harmonic maps from a compact spin Riemann surface with smooth boundary to a general compact Riemannian manifold via a heat flow method when a Dirichlet boundary condition is imposed on the map and a chiral boundary condition on the spinor.

Stephan Luckhaus, University of Leipzig

**Title:** Small angle grain boundaries and piecewise constant rotation fields

**Abstract:** TBA

Xinan Ma, University of Science and Technology of China

**Title:** The Neumann problem for nonlinear elliptic and parabolic equations.

**Abstract:** We shall study the existence of Neumann problem for some geometry elliptic PDE, which include the Hessian equations on strict convex domain, mean curvature equation and special Lagrange equation. Then we study some parabolic corresponding results and the existence of translation solutions on strictly convex domain.

These are the joint work with Chuanqiang Chen, Guohuan Qiu, Peihe Wang, Wei Wei, Dekai Zhang.

Olaf Mueller, Humboldt University Berlin

**Title:** TBA

**Abstract:** TBA

Friedrich Sauvigny, Brandenburgische Technische Universität Cottbus – Senftenberg

**Title:** Multiple solutions for the nonparametric Plateau problem in the Euclidean space  $\mathbb{R}^p$  of arbitrary dimension  $p \geq 3$

**Abstract:** With the aid of Dirichlet's principle, we minimize the area-functional in Section 4 for graphs within  $\mathbb{R}^p$  defined on convex domains  $\Omega \subset \mathbb{R}^2$ ; here the dimension  $p \geq 3$  is arbitrary. In the minimizing sequence we replace the graphs with embeddings of the unit disc via *the uniformization method* (see [10] Chapter XII) from Section 3, and then we substitute them by the harmonic extensions of their boundary values. This is possible within *the class of harmonic embeddings*, due to a beautiful result by H. Kneser (see [9] and [6]). This result has been generalized by J. Jost [4] to harmonic mappings with respect to Riemannian metrics.

We calculate the first and second variation of the area-functional with higher codimensions in Section 2. Already in Section 1 we present *the quasilinear nonparametric minimal surface system* under Dirichlet boundary conditions, where uniqueness does not seem to prevail for  $p \geq 4$ . The constructed solutions above are *stable* in the sense that they possess a nonnegative second variation. However, we indicate by an example of Section 2 how to construct minimal graphs in  $\mathbb{R}^5$  which are unstable. Although Nitsche's uniqueness theorem [8] allows a higher-dimensional analogon via Shiffman's function for polygonal boundaries in  $\mathbb{R}^p$  (see [12]), the nonparametric Plateau problem does not preserve the uniqueness property for higher codimensions, in general.

In Section 5 we prove *a mountain-pass lemma*: An absolute minimizer together with a further strict relative minimizer of the area-functional possess a third minimal graph on the mountain-pass. In comparison with the Morse theory for Dirichlet's integral developed by R. Courant (see [1]), M. Shiffman (see [3], [11]), G. Ströhmer (see [14], [15]), M. Struwe (see [16]), and J. Jost (see [5]), we can consider critical points of the area-functional and apply the continuity theorem of M. Morse and C. Tompkins (see [7]). We refrain from the construction of certain minimizing paths (see [1] and [2]), however, we control the isothermal parameters along the admissible mountain-paths via the theory of nonlinear elliptic systems by E. Heinz (see [10] Chapter XII).

For a complete exposition of our results, we would like referring to our paper [13] dedicated to Professor Dr. Jürgen Jost.

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Ben Sharp, University of Warwick

**Title:** Compactness analysis for minimal hypersurfaces

**Abstract:** We will discuss recent results concerning bubbling analysis and index estimates for minimal hypersurfaces in Riemannian manifolds. The existence theory guarantees that minimal hypersurfaces exist (Almgren, Pitts, Schoen-Simon) and in most cases that they exist in abundance (Marques-Neves, Marques-Neves-Irie). The results presented here are geared towards understanding the relationship between the Morse index and other geometric-analytic qualities of minimal hypersurfaces (topology, diffeomorphism type, total curvature). This will contain joint works with Ambrozio, Carlotto and Buzano.

Guofang Wang, University of Freiburg

**Title:** Hyperbolic masses

**Abstract:**

Ruijun Wu, Max Planck Institute for Mathematics in the Sciences

**Title:** Regularity properties of a nonlinear sigma model with gravitino

**Abstract:** When maps between Riemannian manifolds are coupled with spinorial fields, we are lead to Dirac harmonic maps. This is a reduced version of the full supersymmetric nonlinear sigma model. In this talk we give an introduction to the nonlinear sigma model, in which both the map between Riemannian manifolds and the Riemannian metric get their superpartners in the physical sense. The action functional of this model generalizes the Dirac harmonic map functional. Moreover, the gravitino field gives raise to difficulties of coupled nonlinearity for which we developed new methods to overcome. We will show regularity properties of the critical points of this action, which depends on the gravitino fields and also on the dimensions of the domain manifolds. This is a joint work with J. Jost and M. Zhu.

Yihu Yang, Shanghai Jiao Tong University

**Title:** On complete manifolds with nonnegative Ricci curvature and quadratically nonnegatively curved infinity

**Abstract:** A manifold is said to be of finite topological type if it is homeomorphic to the interior of a compact manifold with boundary. In this talk, I will give a brief introduction to the main results of complete manifolds with nonnegative Ricci curvature and quadratically nonnegatively curved infinity about the finite topological type. This includes some finiteness results under certain conditions of diameter growth (resp. volume growth) and some counterexamples of infinite topology with positive Ricci curvature.

Dong Ye, Université de Lorraine

**Title:** TBA

**Abstract:** TBA

Rugang Ye, University of California, Santa Barbara

**Title:** A Note on Sphere Theorems and the Ricci Flow

**Abstract:** We'll first review sphere theorems proved via the Ricci flow due to Boehme-Wilking, Schoen-Brendle and Brendle. Then we'll present a simplified approach to some of these results.

Hui-Chun Zhang, Sun Yat-sen University

**Title:** Bochner inequality for harmonic maps into singular spaces

**Abstract:** In this talk, we shall introduce a Bochner inequality for harmonic maps into  $CAT(1)$ -spaces, provided its image is contained in a ball with radius  $\rho < \pi/2$ . As an application, we deduce a quantitative gradient estimate for harmonic maps into a non-positively metric space from a manifold  $(M, g)$ , in the term of a lower Ricci bound of  $M$ , and the dimension of  $M$  and the total energy. This answers an open problem proposed by J. Jost in 1998. This is a joint work with Prof. Xiao Zhong and Prof. Xi-Ping Zhu.

Xiao Zhang, Chinese Academy of Sciences

**Title:** The positive energy theorem for asymptotically hyperbolic manifolds

**Abstract:** We provide the positive energy theorem for asymptotically hyperbolic manifolds when the cosmological constant is zero, positive and negative respectively. For the zero cosmological constant, asymptotically hyperbolic manifolds serve as the initial data sets asymptotic to null infinity in asymptotically flat spacetimes, and the positive energy theorem is crucial for studying positivity of the Bondi energy. For the positive or negative cosmological constant, asymptotically hyperbolic manifolds serve as the initial data sets asymptotic to spatial infinity in asymptotically de Sitter spacetimes equipped with hyperbolic coordinates on ends, or in asymptotically anti-de Sitter spacetimes. The difference is that, in the case of zero or positive cosmological constant, the second fundamental forms are asymptotic to hyperbolic metrics while

in the case of negative cosmological constant, the second fundamental forms are asymptotic to zero. In these cases, the Dirac operator can be used to prove the positive energy theorem. The talk is based on the early work of the speaker as well as some joint works with M.X. Luo, N.Q. Xie, Y.H. Wang and Z.B. Liang.

Chunqin Zhou, Shanghai Jiao Tong University

**Title:** Vanishing Pohozaev constant and removability of singularities

**Abstract:** Conformal invariance of two-dimensional variational problems is a condition known to enable a blow-up analysis of solutions and to deduce the removability of singularities. In this paper, we identify another condition that is not only sufficient, but also necessary for such a removability of singularities. This is the validity of the Pohozaev identity. In situations where such an identity fails to hold, we introduce a new quantity, called the Pohozaev constant, which on one hand measures the extent to which the Pohozaev identity fails and on the other hand provides a characterization of the singular behavior of a solution at an isolated singularity. We apply this to the blow-up analysis for super-Liouville type equations on Riemann surfaces with conical singularities, because in the presence of such singularities, conformal invariance no longer holds and a local singularity is in general non-removable unless the Pohozaev constant is vanishing. This is a joint work with Prof. Jürgen Jost and with Prof. Miaomiao Zhu.

Feng Zhou, East China Normal University

**Title:** Existence and symmetry results for some nonlinear biharmonic equations

**Abstract:** We present some recent results concerning the existence, asymptotic behavior and symmetry of solutions to some nonlinear biharmonic equations. We construct solutions with prescribed singular set for a biharmonic equation with the Navier boundary conditions. This is based on joint works with Z. M. Guo, X. Huang, L. Wei, J. C. Wei and D. Ye.

Xi-Ping Zhu, Sun Yat-sen University

**Title:** On four-dimensional manifolds with positive scalar curvature

**Abstract:** It is well known that there exist several differentiable or topological obstructions to compact manifolds admitting metrics of positive scalar curvature. On the other hand, the family of manifolds with positive scalar curvature is quite large since any finite connected sum of them is still a manifold admitting a metric of positive scalar curvature. This talk is concerned with the classification question to this family.

The classical uniformization theorem implies that a two-dimensional compact manifold with positive scalar curvature is diffeomorphic to the sphere or the real projective space. The combination of works of Schoen-Yau and Perelman gives a complete classification to compact three-dimensional manifolds with positive scalar curvature. In this talk we will discuss how to extend Schoen-Yau-Perelman's classification to four-dimension. This is based on the joint works with Bing-Long Chen and Siu-Hung Tang.