

## Program for International Workshop on PDE-Constrained Optimization, Optimal Controls and Applications

09 December (Sunday)	Registration			
Time & Location	Title	Speaker	Chair	
Breakfast	07:30—08:30 (From Monday to Saturday)			
Day One				
09:00—09:15: Opening: Michael Hintermueller and Jun Zou				
10 December (Monday)	09:15—10:00	Semi-smooth Newton method for nonlinear equilibrium conditions and MPEC problems	Kazufumi Ito	Jun Zou
	10:00—10:30	Photo & Tea Break		
	10:30—11:15	Some Progress in Computation of Random/Stochastic Optimal Control	Wenbin Liu	Angela Kunoth
	11:15—12:00	TBA	Xuecheng Tai	
	12:00—13:30	Lunch & Break		
	14:15—15:00	Stabilization and controllability of some free boundary value problems	Marius Tucsnak	Kazufumi Ito
	15:00—15:30	Tea Break		
	15:30—16:15	Regularization Methods for Recovering Conductivity	Xiliang Lu	Fredri Troeltzsch
	16:15—17:00	Accelerated primal-dual methods for PDE-constrained optimization	Christian Clason	
	17:00—17:30	Sparse and switching control for infinite horizon optimal control problems with nonsmooth functionals	Zhiping Rao	
18:00—20:00	Banquet			
Day Two				
11 December (Tuesday)	09:15—10:00	Taylor Expansions for the Hamilton Jacobi Bellman Equation associated to optimal control of Fokker Planck Equations	Karl Kunisch	Michael Hintermueller
	10:00—10:30	Tea Break		
	10:30—11:15	Finite Codimensional Controllability, and Optimal Control Problems with Endpoint State Constraints	Xu Zhang	Ekkehard Sachs
	11:15—12:00	Domain decomposition finite element approximation of optimal control governed by PDEs	Danping Yang	
	12:00—13:30	Lunch & Break		
14:15—15:00	Second Order Adjoints in Optimization	Ekkehard		

			Sachs	Xu Zhang
	15:00—15:30	Tea Break		
	15:30—16:15	Shape derivatives--new perspective and applications to scattering problem	Jingzhi Li	Xuecheng Tai
	16:15—17:00	Nitsche-eXtended finite element methods for optimal control problems of elliptic interface equations	Xiaoping Xie	
	17:00—17:30	Weak Galerkin Method for Electrical Impedance Tomography Inverse Problem	Ying Liang	
	17:30—18:30	Dinner		
Day Three				
12 December (Wednesday)	09:15—10:00	Optimization of Time Delays in a Parabolic Delay Equation	Fredi Troeltzsch	Karl Kunisch
	10:00—10:30	Tea Break		
	10:30—11:15	Asymptotic observability identity for the heat equation in $R^d$	Gengsheng Wang	Marius Tucsnak
	11:15—12:00	Seismic data regularization and imaging	Yanfei Wang	
	12:00—13:30	Lunch & Break		
	Afternoon	Free Discussion 13:30-17:00 As for the sightseeing		
	17:30—18:30	Dinner		
Day Four				
13 December (Thursday)	09:15—10:00	Adaptive Approximations of Parametric PDE-Constrained Control Problems	Angela Kunoth	Wenbin Liu
	10:30—11:00	Tea Break		
	10:30—11:15	TBA	Michael Hintermueller	Gengsheng Wang
	11:15—12:00	Numerical Methods for Stochastic Optimal Control Via FBSDEs	Weidong Zhao	
	12:00—13:30	Lunch & Break		
	14:15—15:00	A posteriori error estimates of the Galerkin spectral methods for fractional optimal control problems	Yanping Chen	Xiliang Lu
	15:00—15:30	Tea Break		
	15:30—16:15	Improved error estimates for semi-discrete finite element solutions of parabolic Dirichlet boundary control problems	Buyang Li	Christian Clason
	16:15—17:00	A penalty scheme and policy iteration for stochastic hybrid control	Yufei Zhang	
	17:00—17:30	Convergence analysis of the Schwarz alternating method for elliptic optimal	Zhiyu Tan	

		control problems		
	17:30—18:30	Dinner		
Day Five				
14 December (Friday)	09:15—10:00	Direct Sampling Methods for General Inverse Problems in an optimization framework	Jun Zou	Danping Yang
	10:00—10:30	Tea Break		
	10:30—11:15	Approximations of tangential boundary control of Stokes equations	Wei Gong	Yanping Chen
	11:15—11:45	An adaptive edge element method and its convergence for an electromagnetic optimal control problem	Bowen Li	
	12:00—13:30	Lunch & Break		
	Afternoon	Departure		
	17:30—18:30	Dinner		
15 December (Saturday)	Departure			

A posteriori error estimates of the Galerkin spectral methods for  
fractional optimal control problems

Yanping Chen

South China Normal University, China

Email: yanpingchen@scnu.edu.cn

Abstract: In this talk, we study the space-time fractional diffusion equations involving a Riemann-Liouville fractional derivative. Both temporal and spatial directions for this equation are discretized by the Galerkin spectral methods, then the spectral accuracy in both space and time are obtained. Consequently, based on the discretization scheme, reliable a posteriori error estimate of the problem is derived. Furthermore, we consider a state constrained optimal control problem governed by time fractional diffusion equation with Riemann-Liouville fractional derivative, which is approximated by the Galerkin spectral method. The optimality conditions are derived, then a posteriori error estimates are obtained for both the coupled state variables and the control variable.

# Accelerated primal-dual methods for PDE-constrained optimization

Christian Clason

Department of Mathematics, University of Duisburg-Essen, Germany

Email: christian.clason@uni-due.de

**Abstract:** The primal–dual hybrid gradient method (PDHGM, also known as the Chambolle–Pock method) has proved very successful for convex optimization problems involving linear operators arising in image processing and inverse problems. We discuss its convergence and acceleration in function space as well as applications to some nonsmooth nonconvex PDE-constrained optimization problems. This is joint work with Stanislav Mazurenko (University of Liverpool) and Tuomo Valkonen (Escuela Politécnica Nacional, Quito).

# Approximations of tangential boundary control of Stokes equations

Wei Gong

Academy of Mathematics and Systems Science, CAS, China

Email: wgong@lsec.cc.ac.cn

**Abstract:** We propose a hybridizable discontinuous Galerkin (HDG) method to approximate the solution of a tangential Dirichlet boundary control problem for the Stokes equations with an  $L^2$  penalty on the boundary control. The contribution of this paper is twofold. First, we obtain well-posedness and regularity results for the tangential Dirichlet control problem on a convex polygonal domain. The analysis contains new features not found in similar Dirichlet control problems for the Poisson equation; an interesting result is that the optimal control has higher local regularity on the individual edges of the domain compared to the global regularity on the entire boundary. Second, under certain assumptions on the domain and the target state, we prove a priori error estimates for the control for the HDG method. In the 2D case, our theoretical convergence rate for the control is superlinear and optimal with respect to the global regularity on the entire boundary. We present numerical experiments to demonstrate the performance of the HDG method. This is a joint work with W. Hu, M. Mateos, J. Singler and Y. Zhang.

Semi-smooth Newton method for nonlinear equilibrium conditions  
and MPEC problems

Kazufumi Ito

North Carolina State University, USA

Email: kito@ncsu.edu

Abstract: We analyze the semi-smooth Newton method for equilibrium condition of the form  $\max_{\alpha \in \{\mathcal{A}\}} \{\mathcal{L}\}^\alpha u + f^\alpha(x, u)$  for  $u$ . Also, we develop and analyze semi-smooth Newton method for Mathematical programming with equilibrium constraints. Motivated applications and numerical tests are presented.

Taylor Expansions for the Hamilton Jacobi Bellman Equation associated  
to optimal control of Fokker Planck Equations

Karl Kunisch

University of Graz and Radon Institute of the Austrian Academy of  
Sciences, Austria

Email: karl.kunisch@uni-graz.at

**Abstract:** Feedback control strategies for the Fokker-Planck equation that speed up convergence to the stationary distributions are investigated. At first, projected Riccati and Lyapunov equation techniques are investigated and their numerical performance is illustrated. Subsequently polynomial approximations of the associated value function are characterized in terms of multilinear forms, which can be obtained as solutions to generalized Lyapunov equations with recursively defined right-hand sides. They form the basis for defining new feedback laws. Their approximation properties are investigated analytically and their performance compared Riccati based laws is investigated numerically. The use of these results in the context of receding horizon control is investigated as well. This is joint work with T. Breiten and L. Pfeiffer.



# Adaptive Approximations of Parametric PDE-Constrained Control

## Problems

Angela Kunoth

University of Cologne, Germany

Email: kunoth@math.uni-koeln.de

Abstract: Optimization problems constrained by linear parabolic evolution PDEs are challenging from a computational point of view: one needs to solve a system of PDEs coupled globally in time and space. Conventional time-stepping methods require an enormous storage. In contrast, adaptive methods in both space and time which aim at distributing the available degrees of freedom in an a-posteriori-fashion to capture singularities are most promising. Employing wavelet schemes for full weak space-time formulations of the parabolic PDEs, we can prove convergence and optimal complexity. Yet another level of challenge are control problems constrained by evolution PDEs involving stochastic or countably many infinite parametric coefficients: for each instance of the parameters, this requires the solution of the complete control problem. Our method of attack is based on the following new theoretical paradigm. It is first shown for control problems constrained by evolution PDEs, formulated in full weak space-time form, that state, costate and control are analytic as functions depending on these

parameters. We establish that these functions allow expansions in terms of sparse tensorized generalized polynomial chaos (gpc) bases. Their sparsity is quantified in terms of  $p$ -summability of the coefficient sequences for some  $0 < p \leq 1$ . Resulting a-priori estimates establish the existence of an index set for concurrent approximations of state, co-state and control for which the gpc approximations attain rates of best  $N$ -term approximation. This entails corresponding sparse realizations in terms of deterministic adaptive Galerkin approximations of state, co-state and control on the entire, possibly infinite-dimensional parameter space. The results were obtained with Max Gunzburger (Florida State University) for the deterministic control problem and with Christoph Schwab (ETH Zurich) for the parametric case.

An adaptive edge element method and its convergence for an  
electromagnetic optimal control problem

Bowen Li

Department of Mathematics, The Chinese University of Hong Kong

Email: [bwli@math.cuhk.edu.hk](mailto:bwli@math.cuhk.edu.hk)

Abstract: An adaptive edge element method is developed for a  $H(\text{curl})$ -elliptic optimal control problem with point-wise constraints on the control. After formulating the optimality system of the control problem, we present the adaptive algorithm in detail and derive the a posteriori error estimation based on the lowest-order edge element discretization for the state and piece-wise constant discretization for the control. With the help of  $L^2$  orthogonal projector and the convergence behavior of the mesh-size functions, we then demonstrate that the sequence of discrete solutions generated by the adaptive algorithm converges strongly to the exact solution. Numerical experiments presented in the last section confirm our theoretical results and indicate the quasi-optimality of the adaptive approach.

Improved error estimates for semi-discrete finite element solutions of  
parabolic Dirichlet boundary control problems

Buyang Li

Department of Applied Mathematics, The Hong Kong Polytechnic  
University

Email: buyang.li@polyu.edu.hk

Abstract: The parabolic Dirichlet boundary control problem and its finite element discretization are considered in convex polygonal and polyhedral domains. We improve the existing results on the regularity of the solutions by establishing and utilizing the maximal  $L^p$ -regularity of parabolic equations under inhomogeneous Dirichlet boundary conditions. Based on the proved regularity of the solutions, we prove  $O(h^{1-1/q_0-\epsilon})$  convergence for the semi-discrete finite element solutions for some  $q_0 > 2$ , with  $q_0$  depending on the maximal interior angle at the corners and edges of the domain and  $\epsilon$  being a positive number that can be arbitrarily small.

Shape derivatives---new perspective and applications to scattering  
problems

Jingzhi Li

Department of Mathematics, Southern University of Science and  
Technology, China

Email: li.jz@sustc.edu.cn

Abstract: This talk studies the “derivative” of solutions of second-order boundary value problems and of output functionals based on them with respect to the shape of the domain. A rigorous approach relies on encoding shape variation by means of deformation vector fields, which will supply the directions for taking shape derivatives. These derivatives and methods to compute them numerically are key tools for studying shape sensitivity, performing gradient based shape optimization, and small-variation shape uncertainty quantification. A unifying view of second-order elliptic boundary value problems recasts them in the language of differential forms (exterior calculus). Fittingly, the shape deformation through vector fields matches the concept of Lie derivative in exterior calculus. This paves the way for a unified treatment of shape differentiation in the framework of exterior calculus. Applications in scattering problems reveals the extraordinary power of the machinery.

Weak Galerkin Method for Electrical Impedance Tomography Inverse  
Problem

Ying Liang

Department of Mathematics, The Chinese University of Hong Kong

Email: [yliang@math.cuhk.edu.hk](mailto:yliang@math.cuhk.edu.hk)

**Abstract:** In this work, we propose a weak Galerkin method for solving the electrical impedance tomography inverse problem based on bounded variation regularization and analyze its convergence. We use the complete electrode model as the forward model and its solution is approximated using the lowest order polynomial bases for weak Galerkin method. The error estimates are studied for the forward problem. We establish the convergence of this weak Galerkin algorithm for the inverse problem, in the sense that the sequence of discrete solutions contains a convergent subsequence to a solution of the continuous bounded variation regularization formulation. We also present numerical results to verify the advantages and efficiency of the algorithm.

Some Progress in Computation of Random/Stochastic Optimal  
Control

Wenbin Liu

University of Kent, UK

Email: W.B.Liu@kent.ac.uk

Abstract: In this work we review some key difficulties and progress in computation of random and stochastic optimal control. We firstly review some progress in adaptive Galerkin methods for the optimal control governed by random PDEs, and, discuss mesh-free adaptive compact RB approximation with numerical tests. We further review some progress in optimal control by SDEs, which are widely used in finance. We present two recent algorithms for the control problems with numerical tests.

# Regularization Methods for Recovering Conductivity

Xiliang Lv

School of Mathematics and Statistics, Wuhan University, China

Email: xllv.math@whu.edu.cn

Abstract: In this talk we propose two regularization models for recovering conductivity. The first one is an  $H^1$  (or TV) regularization for Electrical Impedance Tomography (EIT). The second one is an  $L^2$  regularization for recovering a matrix coefficient. The existence and wellposedness of the regularization functional is proved. The finite element analysis is also provided, and several numerical examples are given to show the efficiency of the proposed method.



Sparse and switching control for infinite horizon optimal control  
problems with nonsmooth functionals

Zhiping Rao

Department of Mathematics and Statistics, Wuhan University, China

Email: zhiping.rao@whu.edu.cn

Abstract: In this talk, a class of infinite horizon optimal control problems involving nonsmooth  $L^p$ -type cost functionals for the controls is discussed. These functionals enhance sparsity and switching properties of the optimal controls. The existence of optimal controls and their structural properties are analyzed on the basis of the first-order optimality conditions. A dynamical programming approach is used for the numerical realizations and the sparse structure of feedback optimal control is discussed in some special cases.

## Second Order Adjoint in Optimization

Ekkehard Sachs

University of Trier, Germany

Email: [sachs@uni-trier.de](mailto:sachs@uni-trier.de)

Abstract: Second order adjoints play an important role in an efficient implementation of fast optimization algorithms. They surfaced over the past decades in various applications under different names. In this talk we take a general viewpoint and derive them in a fairly abstract setting. We point out various applications such as the efficient implementation of the Gauss-Newton method. Furthermore we take a look at a problem in optimization with partial differential equations. We conclude with a special emphasis on an optimization problem in machine learning.

Convergence analysis of the Schwarz alternating method for elliptic  
optimal control problems

Zhiyu Tan

Department of Mathematics, Hong Kong Baptist University

Email: zhiyutan@amss.ac.cn

Abstract: In this paper the Schwarz alternating method for the elliptic optimal control problem is analyzed. We prove the convergence of the method under the maximum norm for both the continuous case and the finite difference discretization case. The Schwarz alternating method is shown to be convergent for the control problem and the convergence rate is bounded from above by the convergence rate for the corresponding elliptic equation, uniformly with respect to the positive regularization parameter. Our numerical experiments agree with the theoretic results very well and show that the method will converge faster when the regularization parameter is smaller. We also give some detailed explanations for this phenomenon.

# Optimization of Time Delays in a Parabolic Delay Equation

Fredi Troeltzsch

Technische Universitaet Berlin, Germany

Email: troeltzsch@math.tu-berlin.de

Abstract: We consider a semilinear parabolic delay differential equation with finitely many time delays. The delays and certain weights for the delay terms should be optimized such that the solution of the partial delay differential equation minimizes the L2-distance to a desired state function. Special emphasis is laid on the partial derivatives of the mapping that associates the solution of the delay equation to the vector of time delays. We improve a result on local differentiability that was proved by J. K. Hale and L.A.C. Ladeira (1993) for a short time horizon. We are able to prove this for any finite time horizon. Based on our differentiability result and adjoint calculus, the gradient of the reduced objective functional can be computed in numerical codes of optimization. Moreover, first-order necessary optimality conditions are derived for the optimal solution. Several numerical examples are presented. This a joint work with Eduardo Casas (Santander) and Mariano Mateos (Gijon)

Stabilization and controllability of some free boundary value  
problems

Marius Tucsnak

University of Bordeaux, France

Email: [marius.tucsnak@u-bordeaux.fr](mailto:marius.tucsnak@u-bordeaux.fr)

Abstract: The first system we consider describes the motion of a rigid body in a viscous incompressible fluid (in 2 or 3 space dimensions). The input of the system is an exterior force acting on the body. The first main result provides feedback laws such that the corresponding closed loop system is well posed and it can be steered to any admissible equilibrium. We next consider a simplified model obtained by replacing Navier-Stokes equations by the viscous Burgers equation in one space dimension. In this case we show that any equilibrium can be attained in finite time. In the second part of the talk we consider stabilization of a Stefan problem. More precisely, we first show that the feedback law proposed in previous work for the one phase Stefan problem can be adapted for the two phases Stefan problem. The main difficulty we tackle is showing that the corresponding closed loop system is well-posed. We also provide some numerical simulations.

# Asymptotic observability identity for the heat equation in $\mathbb{R}^d$

Gengsheng Wang

Center for Applied Mathematics, Tianjin University, China

Email: wanggs62@yeah.net

Abstract: We build up an asymptotic observability identity for the heat equation in the whole space. It says that one can approximately recover a solution, through observing it over some countable lattice points in the space and at one time. This asymptotic identity is a natural extension of the well-known Shannon-Whittaker sampling theorem. According to it, we obtain a kind of feedback null approximate controllability for impulsively controlled heat equations. We also obtain a weak asymptotic observability identity with finitely many observation lattice points. This identity holds only for some solutions to the heat equation.

# Seismic data regularization and imaging

Yanfei Wang

Key Laboratory of Petroleum Resource Research,  
Institute of Geology and Geophysics, Chinese Academy of Sciences,,  
Institutions of Earth Science, Chinese Academy of Sciences,  
Beijing 100029, China

Email: yfwang@mail.iggcas.ac.cn

**Abstract:** We address two main problems in seismic data processing: the first is the compressive seismic acquisition and multi-trace seismic wavefield recovery. To take account of the collective correlation from a given set of testing samples as well as each individual, a matrix minimization model is presented to jointly representing all the testing samples over the coding bases simultaneously. A generalized matrix norm  $\| \cdot \|_{2,p}$  ( $0 < p \leq 1$ ) is employed to measure the interrelation of the multiple samples and the entries of each one. A unified algorithm is developed and the convergence analysis is demonstrated for the range of parameters  $p \in (0,1)$ . The second problem is the seismic imaging. We consider Gaussian beams migration with nonzero initial curvature. Extensive experimental tests are performed to exhibit the efficient performance of the developed methods.

Nitsche-eXtended finite element methods for optimal control problems of  
elliptic interface equations

Xiaoping Xie

School of Mathematics, Sichuan University, China

Email: [xpxie@scu.edu.cn](mailto:xpxie@scu.edu.cn)

**Abstract:** We consider the numerical approximation for two types of optimal control problems governed by elliptic interface equations. We adopt the variational discretization concept to discretize the optimal control problems, and apply extended finite element methods for the elliptic equations to discretize the corresponding state and adjoint equations. Optimal error estimates are derived for the optimal state, co-state and control. Numerical results verify the theoretical results. This is a joint work with Tao Wang and Chaochao Yang.



Domain decomposition finite element approximation of optimal control  
governed by PDEs

Danping Yang

Department of Mathematics, East China Normal University, China

Email: [dpyang@math.ecnu.edu.cn](mailto:dpyang@math.ecnu.edu.cn)

**Abstract:** There have been extensive researches for finite element approximations of optimal control problems in the literatures. In this talk, we will discuss convergence analysis of some domain decomposition methods based upon overlapping or non-overlapping. In the cases with control-constraints, some difficulties different from problems of PDEs or unconstrained control problems are met. We will propose some new technique to treat these problems and give some conditions for selecting the parameters in schemes which lead to better convergent rates or save a lot computational work.

Finite Codimensional Controllability, and Optimal Control Problems with  
Endpoint State Constraints

Xu Zhang

School of Mathematics, Sichuan University, China

Email: zhang\_xu@scu.edu.cn

**Abstract:** Motivated by the study of optimal control problems for infinite dimensional systems with endpoint state constraints, we introduce the notion of finite codimensional (exact/approximate) controllability. Some equivalent criteria on the finite codimensional controllability are presented. In particular, the finite codimensional exact controllability is reduced to deriving a Gårding type inequality for the dual system, which is new for many evolution equations. This inequality can be verified for some concrete problems (and hence applied to the corresponding optimal control problems), say the wave equations with both time and space dependent potentials. Moreover, under some mild assumptions, we show that the finite codimensional exact controllability of this sort of wave equations is equivalent to the classical geometric control condition. (Jointly with Xu Liu and Qi Lü).

A penalty scheme and policy iteration for stochastic hybrid control  
problems

Yufei Zhang

Oxford University, UK

Email: [yufei.zhang@maths.ox.ac.uk](mailto:yufei.zhang@maths.ox.ac.uk)

Abstract: We propose a penalty method for mixed optimal stopping and control problems. The solution and action region of an associated HJB variational inequality are constructed from a sequence of penalized equations, for which the penalization error is estimated. The penalized equation is then discretized by a class of semi-implicit monotone approximations. We further propose an efficient iterative algorithm with local superlinear convergence for solving the discrete equation. Numerical experiments are presented for an optimal investment problem under ambiguity to demonstrate the effectiveness of the new schemes. Finally, we extend the numerical scheme and its convergence analysis to stochastic hybrid control problems involving continuous and impulse controls.

# Numerical Methods for Stochastic Optimal Control Via FBSDEs

Weidong Zhao

School of Mathematics, Shandong University, China

Email: wdzhao@sdu.edu.cn

**Abstract:** In this talk, based on the theories of optimization, stochastic optimal control and forward backward differential equations (FBSDEs), we will introduce some numerical schemes for solving stochastic optimal control. In these schemes, the simplest Euler scheme is used to numerically solve the solutions of the forward stochastic differential equations, and multistep schemes is used to solve the backward stochastic differential equation (BSDE) with high convergence rate. Some stochastic optimal control models, coming from finance and economy, are solved by the schemes. Our numerical results show that our schemes are stable, high accurate, and effective for solving stochastic optimal control problems. This is a joint work with Yu Fu (School of Mathematics and System Science, Shandong University of Science and Technology, Qingdao, Shandong, China, nielf\_fu@sdust.edu.cn) and Tao Zhou (Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China, tzhou@lsec.cc.ac.cn)

Direct Sampling Methods for General Inverse Problems in an  
optimization framework

Jun Zou,

The Chinese University of Hong Kong, Hong Kong

Email: [zou@math.cuhk.edu.hk](mailto:zou@math.cuhk.edu.hk)

**Abstract:** In this talk we shall review the recent developments in direct sampling methods for both nonlinear wave-type and non-wave-type inverse problems. Some general motivations, principles and justifications are discussed for the choices of several key ingredients of the direct sampling methods in an optimization framework, and numerical experiments are presented for various inverse problems.