# Recent Advances in Scientific Computing and Deep Learning

# 科学计算与深度学习最新研讨会

**Dec 18-22, 2023**

TSIMF
清华三亚国际数学论坛

论坛建成十周年
The 10th Anniversary of TSIMF

Sponsored by Northwestern Polytechnical University・School of Mathematics and Statistics

# Conference Brochure

| Dec 18, 2023 (Monday) | | |
|---|---|---|
| Time | Schedule | Chair |
| 08:20-08:30 | Opening Ceremony | |
| 08:30-09:20 | Jie SHEN: A new class of higher-order stiffly stable schemes with application to the Navier-Stokes equations | Andreas PROHL |
| 09:20-10:10 | Xiaoming YUAN: The Balanced Augmented Lagrangian Method for Convex Programming | |
| 10:10-10:40 | Coffee Break | |
| 10:40-11:30 | Andreas PROHL: Numerical Analysis for Optimal Control problems with SPDE constraints | Jie SHEN |
| 12:00-13:30 | Lunch and Break | |
| 14:10-15:00 | Xiangjun TIAN： Handling errors in four-dimensional variational data assimilation by balancing the degrees of freedom and the model constraints: A flow-dependent segmentation 4DVar approach (FS-4DVar) | Quanling DENG |
| 15:00-15:30 | Coffee Break | |
| 15:30-16:20 | Quanling DENG：SoftFEM and softIGA | Xiangjun TIAN |
| 16:20-17:10 | Yulong LU：Diffusion Models: Theory and Applications | |
| 17:30-19:00 | Dinner | |

| Dec 19, 2023 (Tuesday) | | |
|---|---|---|
| Time | Schedule | Chair |
| 08:30-09:20 | Zhiqiang CAI: Neural Network Methods for Scalar Hyperbolic Conservation Laws | Lili JU |
| 09:20-10:10 | Ruchi GUO: Structure Conforming Operator Learning for Inverse Problems: from CNN to Transformer | |
| 10:10-10:40 | Coffee Break | |
| 10:40-11:30 | Lili JU: A Deep Learning Method for the Dynamics of Classic and Conservative Allen-Cahn equations based on Fully-Discrete Operators | Zhiqiang CAI |

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|---|---|---|
| 12:00-13:30 | Lunch and Break | |
| 14:10-15:00 | Jilu WANG：Optimal $L^2$ error estimates of unconditionally stable FE schemes for the Cahn-Hilliard-Navier-Stokes system | Chupeng MA |
| 15:00-15:30 | Coffee Break | |
| 15:30-16:20 | Chupeng MA：A Spectral Generalized Finite Element Method for Multiscale Problems | Jilu WANG |
| 16:20-17:10 | Timo SPREKELER: Numerical homogenization of Hamilton–Jacobi–Bellman equations | |
| 17:30-19:00 | Dinner | |

| Dec 20, 2023 (Wednesday) | | |
|---|---|---|
| Time | Schedule | Chair |
| 08:30-09:20 | Xiaobing FENG: Reinventing Computational Mathematics for High-Dimensional Scientific Computing | Buyang LI |
| 09:20-10:10 | Xiaoping XIE: A fast numerical scheme for fractional Zener viscoelastic model of wave propagation | |
| 10:10-10:40 | Coffee Break | |
| 10:40-11:30 | Buyang LI: Convergent evolving finite element methods with artificial tangential motion for surface evolution | Xiaobing FENG |
| 12:00-13:30 | Lunch and Break | |
| 14:10-17:30 | Free Discussion | |

| Dec 21, 2023 (Thursday) | | |
|---|---|---|
| Time | Schedule | Chair |
| 08:30-09:20 | Guowei WEI: How mathematical AI is transforming biosciences | Juan CHENG |
| 09:20-10:10 | Qianxiao LI: Reduction and Closure of Dynamical Systems using Deep Learning | |
| 10:10-10:40 | Coffee Break | |

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|---|---|---|
| 10:40-11:30 | Juan CHENG: High order entropy stable and positivity-preserving discontinuous Galerkin method for the nonlocal electron heat transport model | Guowei WEI |
| 12:00-13:30 | Lunch and Break | |
| 14:10-15:00 | Xiaoming HE: A decoupled, linear, and unconditionally energy stable finite element method for a two-phase ferrohydrodynamics model | Fei WANG |
| 15:00-15:30 | Coffee Break | |
| 15:30-16:20 | Fei WANG: Randomized Neural Networks Methods for PDEs | Xiaoming HE |
| 16:20-17:10 | Xiaobo YIN: Convergence to local limit of nonlocal models with approximated interaction neighborhoods | |
| 17:30-19:00 | Dinner | |

| **Dec 22, 2023 (Friday)** | | |
|---|---|---|
| Time | Schedule | Chair |
| 08:30-09:20 | Jinchao XU: | Jiwei ZHANG |
| 09:20-10:10 | Jun HU: A Construction of $C^r$ Conforming Finite Element Spaces in Any Dimension | |
| 10:10-10:40 | Coffee Break | |
| 10:40-11:30 | Jiwei ZHANG: An efficient implementation of 3D FEM for nonlocal Poisson problem with different ball approximation strategies | Jinchao XU |
| 12:00-13:30 | Lunch and Break | |
| 14:10-17:30 | Free Discussion | |

# List of Abstracts

## Neural Network Methods for Scalar Hyperbolic Conservation Laws

**Zhiqiang CAI**
Purdue University

Solutions of nonlinear scalar hyperbolic conservation laws (HCLs) are often discontinuous due to shock formation; moreover, locations of shocks are it a priori unknown.

This presents a great challenge for traditional numerical methods because most of them are based on continuous or discontinuous piecewise polynomials on fixed meshes.

As an alternative, by employing it neural network (NN), recently we proposed two NN-based methods for solving HCLs. One is a space-time approach, and the other is an explicit approach emulating the underlying physics. Both the methods show a great potential to sharply capture shock without oscillation, overshooting, or smearing. In one dimension, the second approach is accurate and efficient comparing to existing mesh-based numerical methods.

In this talk, I will give a brief introduction of NN as a class of approximating functions with "moving meshes"and use a simple example to show why the NN is superior to piecewise polynomials on fixed meshes when approximating discontinuous functions with unknown interface. I will then describe both approaches and discuss their pros and cons and related open problems.

## High order entropy stable and positivity-preserving discontinuous Galerkin method for the nonlocal electron heat transport model

**Juan CHENG**
Institute of Applied Physics and Computational Mathematics

The nonlocal electron heat transport model in laser heated plasmas plays a crucial role in inertial confinement fusion (ICF), and it is important to solve it numerically in an accurate and robust way. In this talk, we develop a class of high-order entropy stable discontinuous Galerkin methods for the nonlocal electron heat transport model. We further design our DG scheme to have the positivity-preserving property, which is shown, by a computer-aided proof, to have no extra time step constraint than that required by L2 stability. Numerical examples are given to verify the high-order accuracy and positivity-preserving properties of our scheme. By comparing the local and nonlocal electron heat transport models, we also observe more physical phenomena such as the flux reduction and the preheat effect from the nonlocal model.

## SoftFEM and softIGA

**Quanling DENG**
Australian National University

In this talk, I will present our recent work on SoftFEM (with Alexandre Ern) and softIGA (with Pouria Behnoudfar and Victor Calo). The main idea of softFEM is to reduce the stiffness of the discretised problem by subtracting a least-squares penalty on the gradient jumps across the mesh interfaces from the standard stiffness bilinear form. This penalty bilinear form is similar to the known technique used to stabilize finite element approximations in various contexts. The penalty term is designed to dampen the high frequencies in the spectrum and so it is weighted here by a negative coefficient. We gave a sharp upper bound on the softness parameter weighting the stabilization bilinear form to maintain coercivity for the softFEM bilinear form. We prove that softFEM delivers the same optimal convergence rates. We show numerically that the stiffness reduction ratio scales linearly with p and is of the order of 1 + p/2 for tensor-product meshes and 1 + p/(4-d) for simplicial meshes. SoftIGA extends the main idea of softFEM to the isogeometric element setting.

## Reinventing Computational Mathematics for High-Dimensional Scientific Computing

**Xiaobing FENG**
University of Tennessee

The basic topics of computational mathematics often include interpolation, approximation, numerical differentiation and integration, numerical ODEs and PDEs, optimization, and numerical linear algebra. Classical numerical methods and algorithms were developed for low-dimensional (d=1,2,3) scientific problems because we humans live in a 3-dimensional (or 4-dimensional if spacetime is considered) world. However, recent advances in image processing, financial math, data science, neural networks, and machine learning require solving the abovementioned problems in much higher dimensions. Because of the Curse of Dimensionality (CoD), the classical numerical methods and algorithms become inefficient and/or impractical and/or infeasible for solving those high-dimensional problems. In this talk, I shall use high-dimensional numerical integration and numerical PDEs as examples to present some recent approaches and advances in developing efficient computational methods and algorithms for these two classes of high-dimensional scientific computational problems.

## Structure Conforming Operator Learning for Inverse Problems: from CNN to Transformer

**Ruchi GUO**

The Chinese University of Hong Kong

A deep-learning-based direct sampling method is proposed for solving a class of boundary value inverse problem. A real-time reconstruction is achieved by evaluating the learned inverse operator between carefully designed data and the reconstructed images. An effort is made to give a case study for a fundamental and critical question: whether and how one can benefit from the theoretical structure of a mathematical problem to develop task-oriented and structure-conforming deep neural network? Inspired by direct sampling methods for inverse problems, the 1D boundary data are preprocessed by a partial differential equation-based feature map to yield 2D harmonic extensions in different frequency input channels. Then, by introducing learnable non-local kernel, the approximation of direct sampling is recast to a modified attention mechanism. The proposed method is then applied to electrical impedance tomography, a well-known severely ill-posed nonlinear inverse problem. The new method achieves superior accuracy over its predecessors and contemporary operator learners, as well as shows robustness with respect to noise. This research shall strengthen the insights that the attention mechanism, despite being invented for natural language processing tasks, offers great flexibility to be modified in conformity with the a priori mathematical knowledge, which ultimately leads to the design of more physics-compatible neural architectures.

## A decoupled, linear, and unconditionally energy stable finite element method for a two-phase ferrohydrodynamics model

**Xiaoming HE**

Missouri University of Science and Technology

In this talk, we present numerical approximations of a phase-field model for two-phase ferrofluids, which consists of the Navier-Stokes equations, the Cahn-Hilliard equation, the magnetostatic equations, as well as the magnetic field equation. By combining the projection method for the Navier-Stokes equations and some subtle implicit-explicit treatments for coupled nonlinear terms, we construct a decoupled, linear, fully discrete finite element scheme to solve the highly nonlinear and coupled multi-physics system efficiently. The scheme is provably unconditionally energy stable and leads to a series of decoupled linear equations to solve at each time step. Through numerous numerical examples in simulating benchmark problems such as the Rosensweig instability and droplet deformation, we demonstrate the stability and accuracy of the numerical scheme.

## A Construction of $C^r$ Conforming Finite Element Spaces in Any Dimension

**Jun HU**

Peking University

This talk proposes a construction of $C^r$ conforming finite element spaces with arbitrary $r$ in any dimension. It is shown that if $k \ge 2^{d}r+1$ the space $P_k$ of polynomials of degree $\le k$ can be taken as the shape function space of $C^r$ finite element spaces in $d$ dimensions. This is the first work on constructing such $C^r$ conforming finite elements in any dimension in a unified way.

# A Deep Learning Method for the Dynamics of Classic and Conservative Allen-Cahn equations based on Fully-Discrete Operators

**Lili JU**

University of South Carolina

The Allen-Cahn equation is a well-known stiff semilinear parabolic equation used to describe the process of phase separation and transition in phase field modeling of multi-component physical systems, while the conservative Allen-Cahn equation is a modified version of the classic Allen-Cahn equation that can additionally conserve the mass. As neural networks and deep learning techniques have achieved significant successes in recent years in scientific and engineering applications, there has been growing interest in developing deep learning algorithms for numerical solutions of partial differential equations. In this talk, we present a novel deep learning method for predicting the dynamics of the classic and conservative Allen-Cahn equations. Specifically, we design two special convolutional neural network models, one for each of the two equations, to learn the fully-discrete operators between two adjacent time steps. The loss functions of the two models are defined using the residual of the fully-discrete systems, which result from applying the central finite difference discretization in space and the Crank–Nicolson approximation in time. This approach enables us to train the models without requiring any ground-truth data. Moreover, we introduce an effective training strategy that automatically generates useful samples along the time evolution to facilitate training of the models. Finally, we conduct extensive experiments in two and three dimensions to demonstrate outstanding performance of our proposed method, including its dynamics prediction and generalization ability under different scenarios.

# Convergent evolving finite element methods with artificial tangential motion for surface evolution

**Buyang LI**

Polytechnic University of Hong Kong

A novel evolving surface finite element method, based on a novel equivalent formulation of the continuous problem, is proposed for computing the evolution of a surface in two- and three-dimensional spaces. The method introduces an artificial tangential motion to improve the mesh quality of the approximate surface by minimizing the rate of tangential deformation. Optimal-order convergence of the finite element approximations to the surface evolution is proved.

Numerical examples are provided to illustrate the effectiveness of the proposed method in improving mesh quality of the evolving surfaces.

## Reduction and Closure of Dynamical Systems using Deep Learning

**Qianxiao LI**
National University of Singapore

We discuss some recent work on constructing stable and interpretable macroscopic dynamics from trajectory data using deep learning. We develop a modelling approach: instead of generic neural networks as functional approximators, we use a model-based ansatz for the dynamics following a suitable generalisation of the classical Onsager principle for non-equilibrium systems. This allows the construction of macroscopic dynamics that are physically motivated and can be readily used for subsequent analysis and control. We discuss applications in the analysis of polymer stretching in elongational flow.

## Diffusion Models: Theory and Applications

**Yulong LU**
University of Minnesota Twin Cities

Diffusion models, particularly score-based generative models (SGMs), have emerged as powerful tools in diverse machine learning applications, spanning from computer vision to modern language processing. In the first part of this talk, we delve into the generalization theory of SGMs, exploring their capacity for learning high-dimensional distributions. Our analysis establishes a groundbreaking result: SGMs achieve a dimension-free generation error bound when applied to a class of sub-Gaussian distributions characterized by low-complexity structures. This theoretical underpinning sheds light on the robust capabilities of SGMs in learning and sampling complex distributions.

In the second part of the talk, we shift our focus to the practical realm, demonstrating the application of diffusion models in solving partial differential equations (PDEs). Specifically, we present the development of a physics-informed diffusion model designed for reconstructing high-fidelity solutions from their low-fidelity counterparts. This application showcases the adaptability of diffusion models and their potential to scientific computation.

## A Spectral Generalized Finite Element Method for Multiscale Problems

**Chupeng MA**
Great Bay University

Multiscale partial differential equations, featuring heterogeneous coefficients oscillating across possibly non-separated scales, pose computational challenges for standard numerical techniques. In this talk, I will first review multiscale numerical methods that enable the efficient solution of such problems. Then, I will introduce a multiscale spectral generalized finite element method that builds low-dimensional local approximation spaces based on the singular value decomposition of certain compact operators. Applications of the method and some recent progresses will be presented.

## Numerical Analysis for Optimal Control problems with SPDE constraints

**Andreas PROHL**
University of Tübingen

I compare the 'open loop' strategy ('Pontryagin maximum principle') with the 'closed loop' strategy' ('Riccati equation') concerning their ability to give efficient numerical methods which solve the linear-quadratic control problem with SPDEs. It turns out that the second one is far less computationally demanding, which is quite different to the numerics of deterministic optimal control problems with PDEs.
In the talk, I detail derivation of optimal rates of convergence for both strategies. This is joint work with Yanqing Wang (Southwest University, Chongqing).

## A new class of higher-order stiffly stable schemes with application to the Navier-Stokes equations

**Jie SHEN**
Eastern Institute of Technology

Traditional time discretization schemes are usually based on Taylor expansions at $t_{n+\beta}$ with $\beta\in [0,1]$. However, their ability to deal with very stiff problems are limited by their stability regions. Furthermore, their stability regions decrease as their order of accuracy increase. We show that by using Taylor expansion at $t_{n+\beta}$ with $\beta>1$ as a parameter, we can construct a new class of schemes whose stability region increases with $\beta$, thus allowing us to choose $\beta$ according to the stability and accuracy requirement. In addition, this approach enabled us to solve a long standing problem in the numerical approximation of Navier-Stokes equations. More precisely, no decoupled scheme for the time-dependent Stokes problem with second- or higher-order pressure extrapolation were proven to be unconditionally stable. By choosing suitable $\beta$, we were able to construct unconditionally stable (in $H^1$ norm), decoupled consistent splitting schemes up to fifth-order for the time-dependent Stokes problem.

Then, by combining the generalized SAV approach with the new consistent splitting schemes, we were able to construct unconditionally stable and totally decoupled schemes of second to fourth order for the Navier-Stokes equations, and derive uniform optimal error estimates. We shall also present ample numerical results to show the computational advantages of these schemes.

## Numerical homogenization of Hamilton--Jacobi--Bellman equations

**Timo SPREKELER**

National University of Singapore

In the first part of the talk, we propose and rigorously analyze a mixed finite element method for the approximation of the periodic strong solution to the fully nonlinear second-order Hamilton–Jacobi–Bellman (HJB) equation with coefficients satisfying the Cordes condition. These problems arise as the corrector problems in the periodic homogenization of HJB equations. The second part of the talk focuses on the numerical homogenization of such equations, more precisely on the numerical approximation of the effective Hamiltonian. Numerical experiments demonstrate the approximation scheme for the effective Hamiltonian and the numerical solution of the homogenized problem. This is joint work with Dietmar Gallistl (Jena) and Endre Süli (Oxford).

## Handling errors in four-dimensional variational data assimilation by balancing the degrees of freedom and the model constraints: A flow-dependent segmentation 4DVar approach (FS-4DVar)

**Xiangjun TIAN**

Institute of Atmospheric Physics

For many years, strongly and weakly constrained approaches were the only options for dealing with errors in four-dimensional variational data assimilation (4DVar), with the goal of balancing degrees of freedom and model constraints. Strong model constraints were imposed to reduce the degrees of freedom encountered when optimizing the strongly constrained 4DVar problem, and it was assumed that the models were perfect. The weakly constrained approach attempted to distinguish initial errors from model errors and correct them separately using weak model constraints. Our proposed i4DVar* method exploits the hidden mechanism that corrects initial and model errors simultaneously in the strongly constrained 4DVar. The i4DVar* method divides the assimilation window into several sub-windows, each of which has a unique integral and flow-dependent correction term to treat the initial and model errors simultaneously over a relatively short time period. To overcome the high degrees of freedom of the weakly constrained 4DVar, for the first time we use ensemble simulations not only to solve the 4DVar optimization problem, but also to formulate this method. Thus, the i4DVar* problem is solvable even when there are many degrees of freedom. We show experimentally that i4DVar* provides superior performance at much lower computational cost than existing methods, and is easy to implement.

## Randomized Neural Networks Methods for PDEs

**Fei WANG**

Xi’an Jiaotong University

We propose a novel approach for solving partial differential equations (PDEs) based on randomized neural networks and the Petrov–Galerkin method, which we call the RNN-PG methods. This approach uses randomized neural networks to approximate unknown functions and enables a flexible choice of test functions, such as finite element basis functions, Legendre or Chebyshev polynomials, or neural networks. We apply the RNN-PG methods to various problems, including Poisson problems with primal or mixed formulations, and time-dependent problems with a space–time approach. This talk is based on the work on “Commun. Nonlinear Sci. Numer. Simul. 127, (2023), 107518”, which is adapted from the work originally posted on

arXiv.com by the same authors (arXiv:2201.12995, Jan 31, 2022). The new ingredients include non-linear PDEs such as Burger's equation and a numerical example of a high-dimensional heat equation. Numerical experiments show that the RNN-PG methods can achieve high accuracy with a small number of degrees of freedom. Furthermore, RNN-PG has several advantages, such as being mesh-free, handling different boundary conditions easily, solving time-dependent problems efficiently, and solving high-dimensional problems quickly. These results indicate the great potential of the RNN-PG methods in the field of numerical methods for PDEs. We will also discuss our work on local randomized neural networks with discontinuous Galerkin methods for PDEs (June 11, 2022, arXiv:2206.05577; Comput. Math. Appl. 2024).

## Optimal $L^2$ error estimates of unconditionally stable FE schemes for the Cahn-Hilliard-Navier-Stokes system

**Jilu WANG**

Harbin Institute of Technology

The paper is concerned with the analysis of a popular convex-splitting finite element method for the Cahn-Hilliard-Navier-Stokes system, which has been widely used in practice. Since the method is based on a combined approximation to multiple variables involved in the system, the approximation to one of the variables may seriously affect the accuracy for others.

Optimal-order error analysis for such combined approximations is challenging. The previous works failed to present optimal error analysis in $L^2$-norm due to the weakness of the traditional approach. Here we first present an optimal error estimate in $L^2$-norm for the to convex-splitting FEMs. We also show that optimal error estimates in the traditional (interpolation) sense may not always hold for all components in the coupled system due to the nature of the pollution/influence from lower-order approximations. Our analysis is based on two newly introduced elliptic quasi-projections and the superconvergence of negative norm estimates for the corresponding projection errors. Numerical examples are also presented to illustrate our theoretical results. More important is that our approach can be extended to many other FEMs and other strongly coupled phase field models to obtain optimal error estimates.

## How mathematical AI is transforming biosciences?

**Guowei WEI**
Michigan State University

Mathematics underpins fundamental theories in physics such as quantum mechanics, general relativity, and quantum field theory. Nonetheless, its success in modern biology, namely cellular biology, molecular biology, chemical biology, genomics, and genetics, has been quite limited. Artificial intelligence (AI) has fundamentally changed the landscape of science, engineering, and technology in the past decade and holds a great future for discovering the rules of life. However, AI-based biological discovery encounters challenges arising from the intricate complexity, high dimensionality, nonlinearity, and multiscale biological systems. We tackle these challenges by a mathematical AI paradigm. We have introduced persistent cohomology, persistent spectral graphs, persistent path Laplacians, persistent sheaf Laplacians, persistent hyperdigraph, and evolutionary de Rham-Hodge theory to significantly enhance AI's ability to tackle biological challenges. Using our mathematical AI approaches, my team has been the top winner in D3R Grand Challenges, a worldwide annual competition series in computer-aided drug design and discovery for years. By further integrating mathematical AI with millions of genomes isolated from patients, we uncovered the mechanisms of SARS-CoV-2 evolution and accurately forecast emerging dominant SARS-CoV-2 variants.

## A fast numerical scheme for fractional Zener viscoelastic model of wave propagation

**Xiaoping XIE**
Sichuan University

We propose a scheme for approximating the Mittag-Leffler function by an efficient sum-of-exponentials. The approximation is based on the application of the Gauss-Jacobi quadrature and it converges rapidly. We apply the scheme to a viscoelastic model of wave propagation with mixed finite element methods for the spatial discretization and the Newmark-beta scheme for the second-order temporal derivative. Compared with the traditional L1 scheme for the fractional derivative, our scheme reduces the memory cost to $O(\log N)$ and the computational complexity to $O(N (\log N)^2)$. We provide numerical experiments to verify the theoretical results.

## Convergence to local limit of nonlocal models with approximated interaction neighborhoods

**Xiaobo YIN**

Central China Normal University

Many nonlocal models have adopted a finite and radially symmetric nonlocal interaction domains. When solving them numerically, it is sometimes convenient to adopt polygonal approximations of such interaction domains. A crucial question is, to what extent such approximations affect the nonlocal operators and the corresponding nonlocal solutions. While recent works have analyzed this issue for nonlocal operators in the case of a fixed horizon parameter, the question remains open in the case of a small or vanishing horizon parameter, which happens often in many practical applications and has significant impact on the reliability and robustness of nonlocal modeling and simulations. In this report, we are interested in addressing this issue and establishing the convergence of new nonlocal solutions by polygonal approximations to the local limit of the original nonlocal solutions. Our finding reveals that the new nonlocal solution does not converge to the correct local limit when the number of sides of polygons is uniformly bounded. On the other hand, if the number of sides tends to infinity, the desired convergence can be shown. These results may be used to guide future computational studies of nonlocal problems

## The Balanced Augmented Lagrangian Method for Convex Programming

**Xiaoming YUAN**

University of Hong Kong

We consider the canonical convex minimization problem with both linear equality and inequality constraints, and reshape the classic augmented Lagrangian method (ALM) by better balancing its subproblems. As a result, iterations of the balanced ALM treat the objective function and the coefficient matrix in the constraints separately. Compared with the classic ALM (as well as primal-dual type methods), the balanced ALM may have much easier subproblems and thus can be implemented more easily. We also discuss splitting versions of the balanced ALM for separable convex minimization problems. In particular, these new algorithms can be combined with various deep learning techniques (e.g., PINNs) for solving nonsmooth PDE-constrained optimization problems and others. We report some numerical results when the balanced ALM is applied to solve some academic applications and real-life problems in cloud computing.

# An efficient implementation of 3D FEM for nonlocal Poisson problem with different ball approximation strategies

**Jiwei ZHANG**

Wuhan University

Nonlocality brings many challenges to the implementation of finite element methods (FEM) for nonlocal problems, such as large number of the query and invoke operations on the meshes. Furthermore, the interactions are usually limited to Euclidean balls, so direct numerical integrals often introduce numerical errors, so we have to carefully deal with interactions between the ball with finite elements, such as using ball-approximation strategies. Moreover, the parallel solution of nonlocal problems is more complex to implement than the parallel solution for local problems. Therefore, it is of great significance to propose efficient and general algorithms for solving nonlocal problems. In this talk, we present an efficient representation and construction method of approximate ball based on combinatorial maps and an efficient parallel algorithm for assembly of nonlocal linear systems. In order to conveniently and efficiently deal with numerical integrals over the intersection region of an element with the ball, a new ball-approximation method based on Monte Carlo integrals, the fullcaps method, is also proposed.

# School of Mathematics and Statistics
# Northwestern Polytechnical University

1. Development Prospect

Aiming at the frontiers of international science and technology and crucial national strategic requirement, the root of the frontier basic research of the discipline has been established. The discipline layout focusing on the talent strategy and consolidation of the discipline direction is optimized to cultivate new discipline growth points. The school focuses on the frontier scientific problems in mathematics, statistics and their applications. It carries out the high-level scientific research and high-level talent cultivation. With the guide of high-level scientific research, the school develops high-level international frontier research work, and strengthens international exchanges and collaborations to increase the school's international reputation and influence.

2. Disciplinary Features

After nearly 70 years of development and accumulation, with the aim of "scientific problem-oriented, application problem-driven", The School of Mathematics and Statistics has developed its own integrated discipline system with the characteristics of a strong scientific research strength, a distinctive national defense and teaching scientific research echelon, as well as the firm basic knowledge and highlighted strong application.

3. Major Descriptions

The school currently has 3 undergraduate majors in mathematics and applied mathematics, information and computing science, and statistics. There are 6 master's majors in statistics, pure mathematics, computational mathematics, probability and mathematical statistics, applied mathematics, and operational research and cybernetics, doctoral major and post-doctoral mobile stations in mathematics.

4. Faculty

The School of Mathematics and Statistics has formed a high-level team led by academicians, Distinguished Young Scholars, Cheung Kung scholar, academic leaders and well-known experts with the great introduction and cultivation to talents. The school has 119 full-time staffs including 32 professors, 60 associate professors (38 doctoral supervisors and 83 master advisors) and two foreign instructors. Among them, the school has 3 academicians of the Chinese Academy of Sciences and the Chinese Academy of Engineering，7 National High-level Talent Plan candidates, 3 ministerial New Century Outstanding Talents, 13 provincial and ministerial-level talents. The school has formed a group of high-quality academic staffs, 98.96% of which got doctorate degrees and 87.6% have overseas background.

5. International communications & Collaborations

The school now has two key disciplines of the COSTIND, one national key laboratory, two provincial key laboratories, two international research centers, two international joint laboratories, and one international collaboration base about Operational Research Optimization and Artificial Intelligence International Scientific and Technology. The school has succeeded to apply for two government-sponsored study abroad programs approved by the China Scholarship Council. At the same time, the school has established long-term and effective collaborations with internationally famous universities in the United States, Britain, Germany, France, Belgium, Australia, Italy and other countries in scientific research exchanges and postgraduate joint cultivation.

6. Talent cultivation

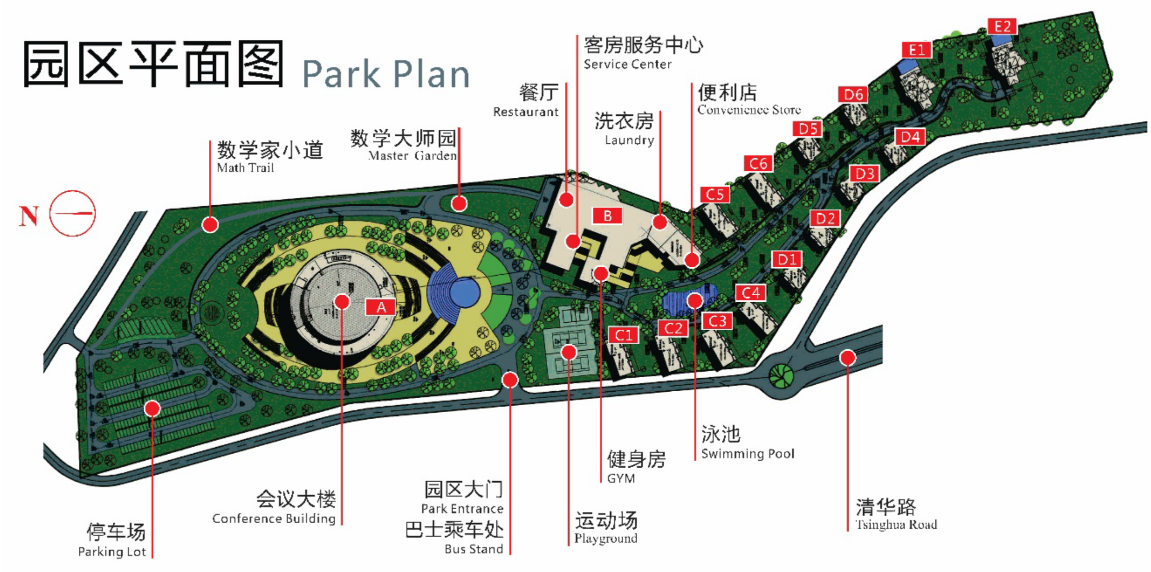
After many years of exploration and practice, the School of Mathematics and Statistics has formed a high-quality talent education system with the target of cultivating outstanding innovative talents of mathematics. In recent years, the School of Mathematics and Statistics has obtained the following representative research achievements. The contributions cover 1 first prize of national education achievement, 6 provincial teaching achievement awards, 6 national first-class courses, 6 provincial first-class courses, 3 excellent teaching teams, 4 educational and teaching reform research projects, 2 provincial excellent textbooks and 15 provincial excellent doctoral dissertations. Up to now, more than 4000 various senior talents are brought up including 3 academicians, several outstanding young scholars, Cheung Kung scholars and many leaders at the provincial and ministerial states.


## Welcome to TSIMF

The facilities of TSIMF are built on a 23-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 29,000 square meter that includes state-of-the-art conference facilities (over 10,000 square meter) to hold many international workshops simultaneously, two reading rooms of library, a guest house (over 10,000 square meter) and the associated catering facilities, a large swimming pool, gym and sports court and other recreational facilities.

Management Center of Tsinghua Sanya International Forum is responsible for the construction, operation, management and service of TSIMF. The mission of TSIMF is to become a base for scientific innovations, and for nurturing of innovative human resource; through the interaction between leading mathematicians and core research groups in pure mathematics, applied mathematics, statistics, theoretical physics, applied physics, theoretical biology and other relating disciplines, TSIMF will provide a platform for exploring new directions, developing new methods, nurturing mathematical talents, and working to raise the level of mathematical research in China.

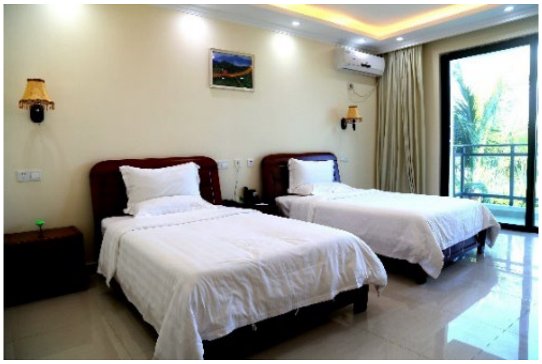
# About Facilities




## Registration


Conference booklets, room keys and name badges for all participants will be distributed at the front desk. Please take good care of your name badge. It is also your meal card and entrance ticket for all events.

## Guest Room



All the rooms are equipped with: free Wi-Fi, TV, air conditioning and other utilities.

Family rooms are also equipped with kitchen and refrigerator.

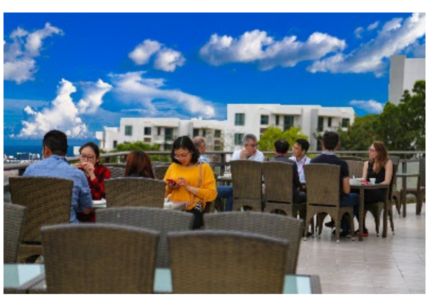
## Library

Opening Hours: 09:00am-22:00pm

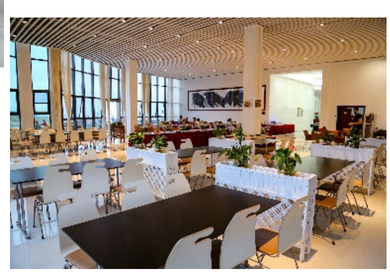
TSIMF library is available during the conference and can be accessed by using your room card. There is no need to sign out books but we ask that you kindly return any borrowed books to the book cart in library before your departure.

In order to give readers a better understanding of the contributions made by the Fields Medalists, the library of Tsinghua Sanya International Mathematics Forum (TSIMF) instituted the Special Collection of Fields Medalists as permanent collection of the library to serve the mathematical researchers and readers.
So far, there are 234 books from 47 authors in the Special Collection of Fields Medalists of TSIMF library. They are on display in room A220. The participants are welcome to visit.

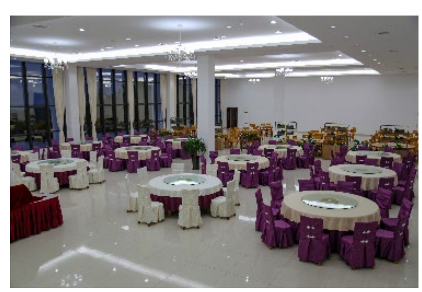
## Restaurant



All the meals are provided in the restaurant (Building B1) according to the time schedule.



| | |
|---|---|
| Breakfast | 07:30-08:30 |
| Lunch | 12:00-13:30 |
| Dinner | 17:30-19:00 |



## Laundry


Opening Hours: 24 hours
The self-service laundry room is located in the Building 1 (B1).

## Gym

The gym is located in the Building 1 (B1), opposite to the reception hall. The gym provides various fitness equipment, as well as pool tables, tennis tables etc.

## Playground

Playground is located on the east of the central gate. There you can play basketball, tennis and badminton. Meanwhile, you can borrow table tennis, basketball, tennis balls and badminton at the reception desk.

## Swimming Pool

Please note that there are no lifeguards. We will not be responsible for any accidents or injuries. In case of any injury or any other emergency, please call the reception hall at +86-898-38882828.

## Free Shuttle Bus Service at TSIMF

We provide free shuttle bus for participants and you are always welcome to take our shuttle bus, all you need to do is wave your hands to stop the bus.

Destinations: Conference Building, Reception Room, Restaurant, Swimming Pool, Hotel etc.


电瓶车路线图 Shuttle Bus Route
Hotel
Reception Room
Restaurant
Conference
Building
A
B
Swimming
Pool
C1
C2
C3
C4
C5
C6
D1
D2
D3
D4
D5
D6
E1
E2

# Contact Information of Administration Staff

**Location of Conference Affairs Office:** ***Room 104, Building A***
Tel: 0086-898-38263896

Conference Manager: Shouxi He 何守喜
Tel:0086-186-8980-2225
Email: hesx@tsimf.cn

**Location of Accommodation Affairs Office: Room 200, Building B1**
Tel：0086-898-38882828
Accommodation Manager: Ms. Li YE 叶莉
Tel: 0086-139-7679-8300
Email: yeli@tsimf.cn

**Director Assistant of TSIMF**
Kai CUI 崔凯
Tel/Wechat: 0086- 136-1120-7077
Email :cuikai@tsimf.cn

**Director of TSIMF**
Prof.Xuan GAO 高瑄
Tel: 0086-186-0893-0631
Email: gaoxuan@tsinghua.edu.cn